A GLOBAL IONOSPHERE-PLASMASPHERE MODEL: APPLICATIONS TO GPS

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ABSTRACT

With the recent advent of GPS satellites as a research tool in studying the ionized regions that surround the Earth a need has arisen for a simple yet accurate model for plasmasphere, the region above the ionosphere. A dynamic model based on diffuse equilibrium will be described, which is combined with the International Reference Ionosphere model to produce a global ionosphere-plasmasphere model. Some preliminary results from the model are presented and discussed.

INTRODUCTION

The extension of plasma from the ionosphere up the Earth's closed magnetic field lines forms the plasmasphere. One of the main methods to study these regions is to investigate their electron densities by observing the Total Electron Content (TEC) along a ray path between a satellite and a ground receiver. The TEC is the total number of electrons in a column with a cross sectional area 1 m^2 along the ray path.

The Global Positioning System (GPS) Satellites are one major source of this type of data, with large data sets available from numerous observation sites across the world taken over many years. GPS satellites orbiting at an altitude of 20,200 km, thus a radio signal received on Earth must generally have propagated through the plasmasphere and the underlying ionosphere. While the plasmasphere's electron density is on average one to two orders of magnitudes less than that of the ionosphere, the greater distance travelled through the plasmasphere means that the total amount of plasma traversed by the signal is roughly of the same order of magnitude. Consequently the plasmasphere's effect on a GPS signal needs to be considered at all times.

An increasing amount of the ionospheric electron density research being conducted at La Trobe University is based on GPS satellite data. It has therefore become important to understand the effects that the plasmasphere's electron densities have on these GPS measurements, so that they can be correctly interpreted. To study these effects a global electron density model of the ionosphere and plasmasphere has been developed.

In the past many computer-based models of the plasmasphere have been created, the FLIP model (Torr et al. 1990) and the Sheffield model (Bailey and Balan 1996) being two current examples. Their primary aim is to model the complex physical processes of the plasma flows along the magnetic field lines. The aim of our research was to produce a simple yet accurate global electron density model that could be run on a desktop PC. The major goal of the model is to allow a user to specify a ray path between the two points and have the model quickly and accurately determine the electron density at each point along the ray path. With this information parameters such as TEC can easily be calculated.

THE PLASMASPHERE

The dominant ion species in the plasmasphere is hydrogen, hence the plasmasphere has in the past been referred to as the protonsphere. These H^+ are produced through the reaction:

$$O^+ + H \Leftrightarrow O + H^+ \tag{1}$$

which is accidentally energetically resonant and proceeds rapidly in both directions. The production rate of H^+ in $m^{-3}s^{-1}$ is given by:

$$P = 2.5x10^{-17} T_n^{1/2} n(H)n(O^+) \tag{2}$$

and the loss rate by:

$$L = 2.2x10^{-17} T_i^{1/2} n(O)n(H^+) \tag{3}$$

where T_n is the neutral temperature, T_i is the ion temperature and n(#) is the number density of the given ion or neutral atom species. Equation (1) predominately occurs in the upper ionosphere and lower plasmasphere, due to the rapid drop in the oxygen (ion and neutral) densities, which retards both production and loss at higher altitudes.

The H^+ flow outwards along the magnetic field lines and becomes trapped at lower latitudes in the closed magnetic field lines forming the plasmasphere. In the polar regions where the field lines are open the H^+ do not become trapped and flow out into space, producing what is called the 'polar wind'. The plasmasphere extends out to 3 to 6 Earth radii (some 20,000 km to 40,000 km), with plasma densities of the order 10^8 - 10^9 electrons/m^3.

AN IONOSPHERE/PLASMASPHERE GLOBAL ELECTRON DENSITY MODEL

The global electron density model of the ionosphere/plasmasphere that has been developed is based on diffusive equilibrium, with chemical equilibrium used at low altitudes. The model requires three outside major sources of data:

1. Electron and ion temperatures.
2. The neutral temperature, oxygen and hydrogen densities.
3. Ionospheric electron density.

The required neutral parameters are obtained from the MSIS-90 model (Hedin 1991, Hedin 1986). The ionospheric electron densities are obtained from the IRI95 model (Bilitza 1995). Both these models can be down loaded as FORTRAN programmes from the NSSDS Web Site (NSSDC 1999). The electron and ion temperatures are calculated using a modified version of the upper ionosphere and plasmasphere temperature model published by Titheridge (1998).

Because the plasma is constrained to move along magnetic field lines the diffusive profiles must be calculated along the field lines. The diffusive equilibrium equations used are given in Titheridge (1972) and are reproduced below (note the corrected scale height formula):

$$H_j = \left(\frac{kT_i}{m_j g}\right)\left(1 - \frac{m_a T_e}{m_j T_t}\right)^{-1} \tag{4}$$

where H_j is the j'th ion scale height, k is the Boltzmann constant, T_i is the ion temperature, m_j is the j'th ion mass, g is gravity, m_a is the mean ion mass, T_e is the electron temperature and $T_t = T_i + T_e$. Taking n_{j0} as the number density at height h_0 with total temperature T_{t0}, the density n_j at the height $h_0 + \Delta h$ with total temperature T_t is:

$$n_j T_t = n_{j0} T_{t0} \exp\left(-\frac{\Delta h}{H_j}\right) \tag{5}$$

As Titheridge noted, "the form given above is particularly suitable for numerical integration". If the density of a given ion species at a certain base height is known, then using incremental steps the density can be calculated at some greater height. This is only possible if the temperatures are known at each step and no chemical ion production or loss is occurring. Because of the large amount of production and loss in the ionosphere, diffusive equilibrium profiles are not always accurate and chemical equilibrium profiles obtained from Equations (2) and (3) need to be considered.

A simple diffusive model would consider only O^+ and H^+ and use Equations (4) and (5), starting from a base height in the ionosphere where chemical equilibrium is assumed. The resulting O^+ profile along the field line will generally agree with those observed, however the H^+ profile produced will only agree in the case where the magnetic field line (more correctly the magnetic flux tube) is saturated. This occurs when the H^+ production and loss given by Equations (2) and (3) are equal and there is no net change in the number of H^+ in the flux tube.

The plasmasphere is rarely saturated however, especially at higher latitudes. This is due to diurnal variations in temperature, the ionosphere and neutral densities, and magnetic storms that empty the outer plasmasphere. The result is a highly dynamic plasmasphere with plasma consisting predominantly of H^+ flowing up into the plasmasphere or flowing down into the ionosphere. A simple model that projects up from the ionosphere assuming chemical and diffusive equilibrium will generally over estimate the correct plasmaspheric density by one to three orders of magnitude. This approach also has the problem that H^+ profiles projected up from northern and southern hemispheres along a given field line will rarely have matching densities at the magnetic equator.

A solution to these problems was suggested by Phil Richards (author of the FLIP model, private communication 1996), and is the basis of the model has been developed. If the total hydrogen ion content of a flux tube is known then to a good approximation the equatorial density is equal to the total content divided by the total volume of the flux tube. This approximation is generally correct to within 5% and is due to the majority of the volume been centred around the magnetic equator, where the H^+ density is only slowly varying. Starting from this equatorial density the H^+ density profile is project down the field line using diffusive equilibrium into both hemispheres. This automatically solves the problem of equatorial mismatch when the profiles are projected up the field lines from the two hemispheres. If the flux tube is depleted due to magnetic storm activity this is also taken into account by the lower equatorial density. Note that the O^+ profile is still obtained by projecting up from the underlying ionosphere.

A problem this approach does create is that as the profiles reach down into the ionosphere need to be matched up with the chemical equilibrium profiles. A desire to have close agreement between the model under development and the FLIP model served as a guide to the profile that the H^+ should have. A method has been developed to smoothly join the two profiles based around an equation derived by Richards and Torr (1985) that estimates the height at which chemical loss is equal to diffusive loss. The corrected equation (Rasmussen et al. 1993) is given below:

$$z_0 = z_r - \left(\frac{H_1 H_2}{H_1 + H_2}\right) \ln\left(\frac{7.5\text{x}10^{19} T^2}{H^2 n_r(O^+) n_r(O)}\right) \tag{4}$$

where z_0 is the required height, z_r is some reference height, H is the diffusion scale height, H_1 is the O^+ scale height, H_2 is the O scale height, T is the ion temperature, $n_r(O^+)$ is the O^+ number density and $n_r(O)$ is the O number density, both at the reference height r. All heights are in km, and densities are in m^{-3}. Based on this equation a weighting function has been derived to smoothly join the chemical and diffusive H^+ profiles. An example resulting profile compared to a FLIP model profile is shown in Figure 1.

Figure1 shows the good agreement in this case between the FLIP model's H^+ profile and our model. At 400–500 km chemical equilibrium dominates, while the diffusive equilibrium profile projected down the field line controls the H^+ above 1000 km. The disagreement at higher altitudes between the models is due to the different O^+ profiles and differences in the model's ion and electron temperatures.

With the ability to correctly reproduce the H^+ profiles the time evolution of the H^+ content of the field line can be determined. Since at any one given time the neutral densities and ion densities are known, both the production and loss for the given field line can be calculated using Equations 2 and 3. Assuming the density profiles do not change over some small time increment, the net change in the H^+ content of the field line can be calculated. Time can be advanced by the time increment, a new equatorial density calculated as before and the process repeated. This allows the time development the H^+ content of a magnetic field line to be modelled.

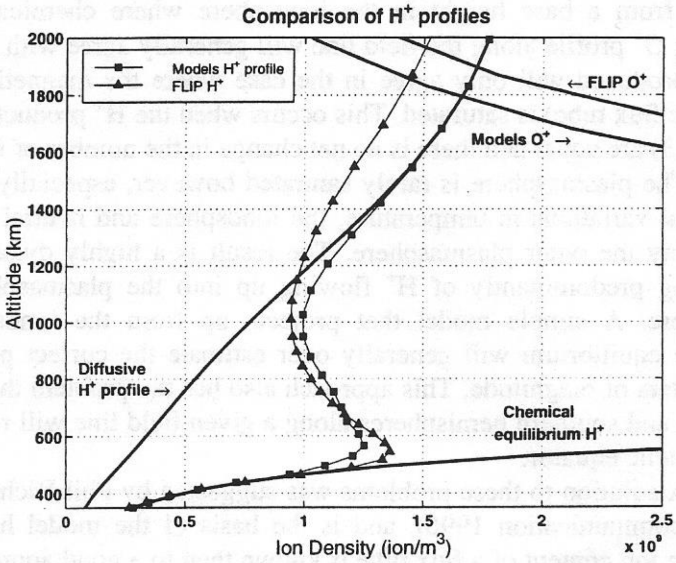

Figure 1: FLIP model and La Trobe model H+ profiles

The final dynamic aspect of a model of the plasmasphere that needs to be considered is the motion through space of the individual magnetic field lines, or more correctly the motion of the plasma aligned along a magnetic field line in the corresponding magnetic flux tube. At low magnetic latitudes the plasma 'sticks' to the field lines and co-rotates with the Earth. At higher latitudes the electric fields in the Earth's magnetosphere cause the plasma to move from one magnetic field line to another, with the magnitude of this motion being directly related to the current solar activity. During periods of large solar magnetic storms the outer plasmasphere is torn away from the Earth by the increased strengths of these electric fields and lost into the magnetosphere.

A simple model to describe this motion is to combine a rotational potential centred on the Earth with a uniform dawn to dusk potential across the magnetosphere (see for example Wolf 95). The resulting potential ϕ is:

$$\phi = -E_0 r \sin(\theta) - \frac{\omega_E B_0 R_E^3}{r} \tag{6}$$

where E_0 is the magnetospheric electric field, r is the radial distance from the centre of the Earth, θ is the magnetic longitude, ω_E is the Earth's angular velocity, B_0 the magnetic field strength at Earth's surface and R_E is the Earth radii. The velocity of the flux tubes is the equatorial plane is then given by:

$$v = \frac{B \times \nabla\phi}{B^2} \tag{7}$$

where the magnetic field is assumed to be given by a magnetic dipole.

EXAMPLES OF RESULTS FROM THE MODEL

Because the model has only been recently completed, the results that will be presented here are preliminary. They do however indicate the type of results that the model is capable of producing, noting that its primary goal is to calculate the electron density at any point surrounding the Earth out to a radius of 12 Earth radii.

Figure 2 shows an example of 'snapshot' of the global ionosphere/plasmasphere equatorial electron density produced by the model. Several features to note are the plasmapause, with its 1 - 2 order of magnitude drop in electron density and the plasmasphere's evening bulge at 18 MLT.

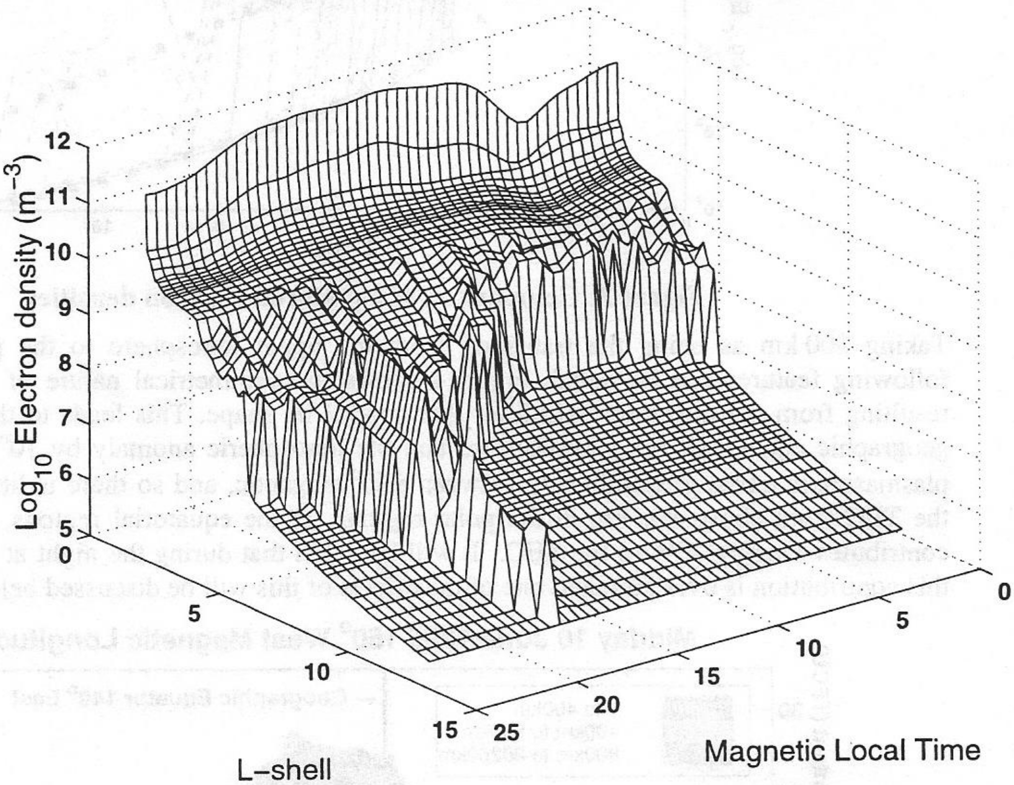

Figure 2: Example of global equatorial electron density profile

Figure 3 plots equatorial electron densities from Figure 2 against their L-shell. For comparison, the saturated equatorial profile obtained from the empirical model by Carpenter and Anderson (1992) is also plotted, as well as the densities from their model for the region beyond the plasmapause, the plasmatrough. As Figure 3 shows, there is good agreement between the two models.

With the ability to calculate electron densities along a ray path, it is straightforward to calculate the TEC. An example of this shown in Figure 4 where the TEC has been calculated in the vertical direction between the North and South magnetic poles at150° magnetic longitude. Unlike direct satellite observations where only the total TEC can be measured, the contributions to the TEC from various altitude bands can be calculated. The examples shown in Figure 4 are for solar minimum, at midday and midnight, with 20,200km chosen as it corresponds to the altitude of the GPS satellites. Note that in these examples that the TEC above 20,200km is at the most 0.15 TECU which is insignificant.

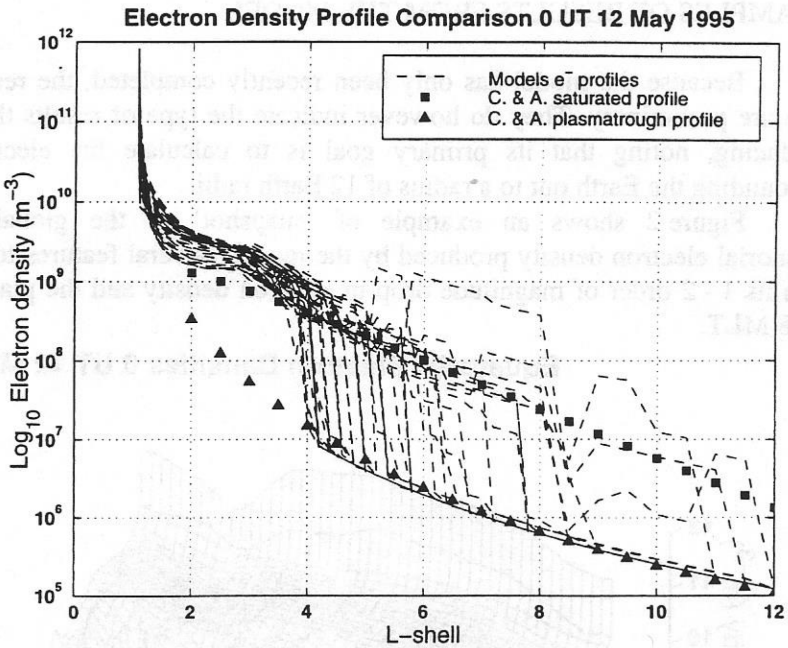

Figure 3: Comparison of equatorial electron densities

Taking 800 km as being the transition from the upper ionosphere to the plasmasphere, the following features can be noted. Figure 4 shows the symmetrical nature of the plasmasphere resulting from the magnetic field lines that control its shape. This leads to the offset from the geographic equator of the plasmasphere and the ionospheric anomaly by 10° in this case. The plasmasphere exists predominately between $\pm 50^{\circ}$ magnetic, and so there is little contribution to the TEC from above 800 km in the polar regions. In the equatorial regions, the plasmasphere contributes roughly 30% to the TEC. It will be noted that during the night at southern latitudes this contribution is over 50% and one of the effects of this will be discussed below.

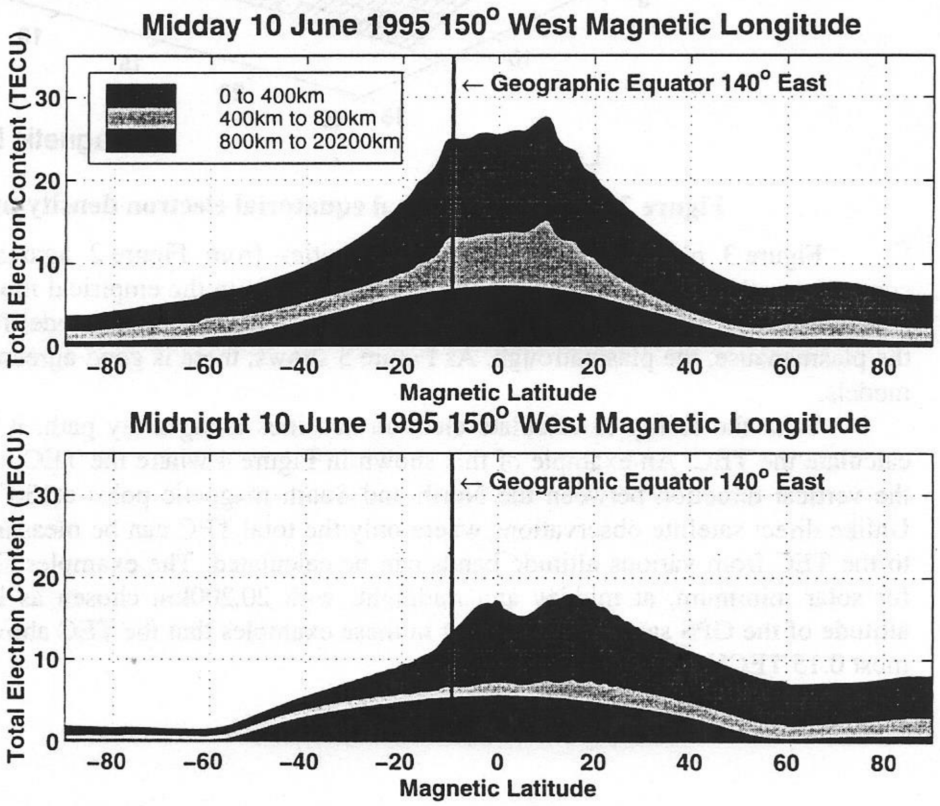

Figure 4: Example of Vertical TEC Calculation.

Knowledge of the electron densities along a ray path allows another ionospheric parameter to be calculated, the mean ionospheric height. This is the altitude of the point along a ray path where half the TEC is above the point and half is below. It is required for example when converting from slant TEC to vertical TEC, and is normally assumed to be a constant altitude of 400 km. In Figure 5 the mean ionospheric height has been calculated along the same magnetic longitude as Figure 4. Figure 5 shows that in this example assuming a constant mean ionospheric height of 400 km is a good approximation, except during the night at southern latitudes where the correct value is over 1000 km. This is caused by the plasmasphere contributing over 50% to the TEC in this region, has shown in Figure 4. Consequently any vertical TEC calculated from slant TEC using the 400 km approximation will be in error.

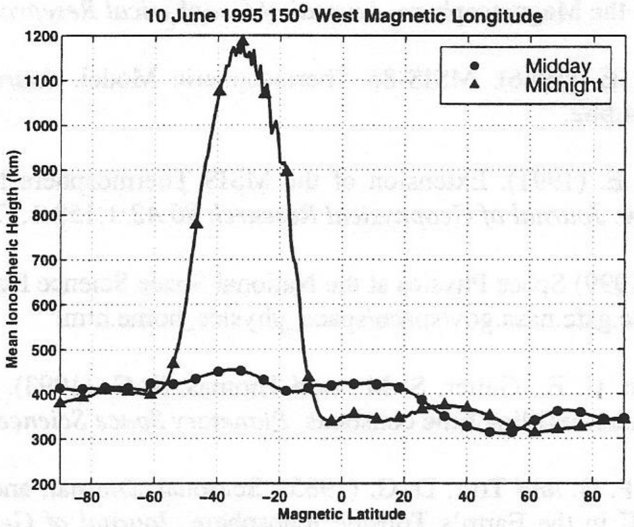

Figure 5: Example of Mean Ionospheric Height

Other results that can be obtained from the model include studying the plasmasphere's recovery after magnetic storms and the effects this would have on observed TEC measurements from GPS satellites. The length of time taken by the plasmasphere to recover can be estimated by observing the refilling of the magnetic field lines. With the ability to calculate the plasmaspheric contribution to the observed TEC from a GPS satellite, it is possible to remove it so that the ionospheric contribution can be separately determined. This correction will allow more accurate studies of the ionosphere to be undertaken using GPS satellite signals. The model will be used to study, for example, the expected TEC along ray paths between two satellites, such as the Australian FedSat satellite and the GPS constellation.

CONCLUSION

A global electron density model of the ionosphere/plasmasphere has been developed at La Trobe University, which allows the calculation of electron densities at any point out to 12 Earth radii from the Earth. This allows the densities along a given ray path can be calculated, from which parameters such as the Total Electron Content and mean ionospheric height can be determined. Knowledge of these parameters is important in allowing the correct interpretation of GPS satellite signals received by ground stations that have passed through the ionosphere and plasmasphere.

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Speaking for the organizing committee, I am pleased that you have considered IES99 as an appropriate forum for presenting your latest ionospheric data and results. I also hope you take the time to renew scientific acquaintances in the furtherance of ionospheric research. As solar cycle 23 winds toward its projected zenith in the year 2000, we can expect to have an enjoyable, informative, and exciting time.

Sincerely,

John Goodman

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