Vacuum Polarization in the Lee and Yang Theory of Charged Spin One

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Summary The radiative correction, to order e^2, to the photon propagator in the Lee and Yang theory of charged spin one bosons, is calculated in a dispersion-theoretic manner. Some general aspects of the ξ-limiting formalism are discussed. A useful matrix treatment of Lorentz tensors is presented in an Appendix

1 Introduction

The theory of charged bosons of spin one devised by Lee and Yang[(1)] has, despite its apparent weaknesses[(2)], not yet been supplanted, so that it is of interest to perform a purely quantum electrodynamical calculation in this formalism[(3)]. To this end we herein present in Section 2 a dispersion-theoretic calculation of the lowest order radiative correction to the photon propagator in the ξ-limiting formalism. As anticipated, the derivative of the polarization for zero momentum squared, viz $\Pi(0)$, diverges as the parameter $\xi \to 0$. In Section 3 we discuss the value arrived and comment on a feature of the Lee and Yang theory that seems to emerge from this calculation: namely that, unlike the "good" theories of charged spin zero and charged spin$\frac{1}{2}$ there is no close correspondence between the qualitative features of the c-number and of the second quantised theory.

In Appendix I we detail a useful matrix treatment of tensor quantities.

2 Calculation of $\Pi'(0)$

In the Lee and Yang theory of charged bosons of spin one [(1)] the charged particle propagator is taken to be

$$S^{LY}_{\beta\alpha}(p) = S^1_{\beta\alpha}(p) + S^0_{\beta\alpha}(p) \tag{2.1}$$

where

$$S^1_{\beta\alpha}(p) = iN^1_{\beta\alpha}(p)/(p^2 + M_1^2 - i\epsilon) \ ; \ S^0_{\beta\alpha}(p) = iN^0_{\beta\alpha}(p)/(p^2 + M_0^2 - i\epsilon) \quad (2.2)$$

$$M_1 = m \quad ; \quad M_0 = \xi^{\frac{1}{2}} m \quad (2.3)$$

m being the mass of the spin one particle, while Lee and Yang specify that ξ is to be small. The numerator terms are explicitly

$$N^1_{\beta\alpha}(p) = \delta_{\beta\alpha} + p_\beta p_\alpha/m^2 \quad ; \quad N^9_{\beta\alpha} = p_\beta p_\alpha/m^2 \quad (2.4)$$

The 3-vertex of this theory has the value

$$eV_{\mu;\beta\alpha}(p'', p') = -ir\delta_{\beta\alpha}(p''+p')_\mu - ie\delta_{\alpha\mu}(-\kappa p''+p'+\kappa p'-\xi p'')_\beta - ie\delta_{\beta\mu}(-\kappa p'+p''+\kappa p''-\xi p')_\alpha \quad (2.5)$$

a semi-colon being utilised to separate the photon index from the indices of the incoming and outgoing charged particles. The 4-vertex does not occur in our calculation.=, so that the corrected propagator is

$$D_{\mu\nu}(k) = -i(\delta_{\mu\nu}/k^2) - i(\delta_{\mu\nu} - k_\mu k_\nu/k^2)\Pi(k^2) \quad (2.6)$$

The Lehman-Kallen[4] dispersion relation for the vacuum polarization $\Pi(k^2)$ is

$$\Pi(k^2) = \frac{-k^2}{\pi} \int_0^\infty \frac{ds \ \mathrm{Im}\,\Pi(-s)}{s(s + k^2 - i\epsilon)} \quad (2.7)$$

We note that $\mathrm{Im}\,\Pi(k^2) = \pi k^2 \rho_3(-k^2)$, where the spectral weight ρ_3 would be positive definite in a theory where Hilbert space is endowed with a positive definite metric. In this calculation we determine

$$\Pi'(0) = [\mathrm{Lim}\ k^2 \to 0]\ \Pi(k^2)/k^2 = -\frac{1}{\pi} \int_0^\infty \frac{ds \ \mathrm{Im}\,\Pi(-s)}{s^2} \quad (2.9)$$

Because of the relatively large masses of the known vector mesons, a knowledge of the value of $\Pi'(0)$ is adequate for any investigation of possible spectroscopic detection of this component of vacuum polarization.

To order e^2, the vacuum polarization is

$$\Pi = \Pi^{11} + \Pi^{01} + \Pi^{10} + \Pi^{00} \quad (2.9)$$

where

$$\Pi^{ab} = \frac{-i}{3k^2} \frac{e^2}{(2\pi)^4} \int d^4p' d^4p'' \delta(k-p'+p'')\delta_{\mu\nu} \mathrm{Tr}[S^a(p')V_\mu(p', p'')S^b(p'')V_\nu(p'', p')] \quad (2.10)$$

In this formula, and subsequently below, we have suppressed the charged particle indices, $S^a(p)$, $V_\mu(p', p'')$ being regarded as 4x4 matrices, as detailed in Appendices 1 and 2. The operator Tr involves taking the trace of these matrix expressions.

As states containing an odd number of spinless particles have negative norms, the denomnators of both $S^a(p)$ have the comon form $(p^2 + M_a^2 - i\epsilon)$; we can therefore at once apply the analysis of Cutkosky [(5)] to determine the jump discontinuity across the branch cuts of each $\Pi^{ab}(k^2)$, the branch points being at the Landau[6] position $k^2 = -(M_a + M_b)^2$. On making the replacement

$$(p^2 + M_a^2 - i\epsilon)^{-1} \to 2\pi i \delta_p[p^2 + M_a^2] \qquad (2,11)$$

where the subscript p on the delta function means that only the proper root is to be taken, the branch-cut discontinuity is found to be

$$2i\mathrm{Im}\,\Pi^{ab}(k) = -\frac{ie}{12\pi^2 k^2}\int dp''\, p'\, dp''\, \delta(k - p' + p'')\delta_p[p'^2 + M_a^2]\delta_p[p''^2 + M_b^2]Tr^{ab} \qquad (2.12)$$

Thus

$$\mathrm{Im}\,\Pi^{ab}(k) = \frac{-e^2 m^2}{48\pi k^2} f^{ab}(k)\ \mathrm{Tr}^{ab}\theta[-k^2 - (M_a + M_b)^2] \qquad (2.13)$$

where

$$f^{ab}(k^2) = [1 + 2\frac{(M_a^2 + M_b^2)}{k^2} + (\frac{M_a^2 - M_b^2}{k^2})^2]^{\frac{1}{2}} \qquad (2.14)$$

and

$$\mathrm{Tr}^{ab}(k^2) = m^{-2}\ \mathrm{Tr}[N^a(p', p'')M^b(p'')V_\mu(p'', p')] \qquad (2.15a)$$

the trace, i.e., sum over vector indices, being evaluated on the mass-shell

$$k = p' - p''\ ,\ p'^2 + M_a^2 = 0\ ,\ p''^2 + M_b^2 = 0 \qquad (2.15b)$$

The $\mathrm{Tr}^{ab}(k^2)$ are evaluated in Appendix II, where it is found that

$$\mathrm{Tr}^{11}(m^2 x) = \frac{1}{4}\kappa^2 x^3 - (1 + 3\kappa)x^2 \qquad (2.16)$$

and

$$\mathrm{Tr}^{00}(m^2 x) = \frac{1}{4}\kappa^2 x^3 + (\kappa^2\xi^{-1} - \kappa)x^2 + (-4\kappa\xi^{-1} + 1)x + 4\xi^{-1} \qquad (2.17)$$

The expression for TR^{01} is rather lengthy and has been placed at the end of Appendix II.

We note that Im $\Pi^{11}(k)$ has exactly the same value as is given in the usual "unrenormalisable" theory of charged particles of spin one, mass m, anomalous moment κ [Lee and Yang[(1)] proved that for $\kappa \neq 0$ the Dyson-Wick canonical formalism leads to additional non-covariant vertices and hence further divergent terms]. Likewise, for $\kappa = 0$, Im $\Pi^{00}(k)$ has exactly the magnitude and the usual (-) sign of the contribution to vacuum polarization ascribable to charged spin zero bosons of mass M_0 . On the other hand Im $\Pi^{01}(k)$ is positive, and this is the counter term which leads to a convergent theory. The asymptotic value of Im $\Pi(k)$ may be readily found: it is negative and constant.

$$\text{Im}\,\Pi(k) \approx \frac{-e^2}{48\pi}[3(1-\kappa^2]\xi^{-1} - 5 - 12\kappa - 3\kappa^2] \;\; as \;\; \xi^{-1}m^2 << -k^2 \to \infty \quad (2.18)$$

It follows, as anticipated from crude counting of powers of momentum in equations (2.9), (2.10), that the dispersion integral of equation (2.7) does converge. As however the various contributions $\Pi^{ab}(k^2)$ to $\Pi(k^2)$ do not separately converge, we find it convenient to write

$$\widehat{\Pi}'(0) = [\lim\lambda \to 0]\frac{e^2}{48\pi^2 m^2}[\widehat{\Pi}^{11}(\lambda) + 2\widehat{\Pi}^{01}(\lambda) + \widehat{\Pi}^{00}(\lambda)] \quad (2.19)$$

where

$$\Pi^{ab}(\lambda) = -m^4 \int_{(M_a+M_b)^2}^{M_0^2\lambda^{-1}} \frac{dS}{S^3} f^{ab}(-S)\text{Tr}^{ab}(-S) \quad (2.20)$$

These elementary integrals are readily evaluated. One takes first the limit $\lambda \to 0$, and subsequently takes ξ small to get

$$\widehat{\Pi}^{11}(\lambda) = \frac{1}{4}\kappa^2\xi^{-1}\lambda^{-1} - (1+3\kappa-\frac{1}{2}\kappa^2)\log\lambda - (1+3\kappa-\frac{1}{2}\kappa^2)\log\xi - \frac{71}{30} - 8\kappa - \frac{7}{6}\kappa^2 \quad (2.21)$$

$$\widehat{\Pi}^{01}(\lambda) = -\frac{1}{2}\kappa^2\xi^{-1}\lambda^{-1} - (\frac{3}{2}\kappa^2\xi^{-1} - 1 - 4\kappa + \frac{1}{2}\kappa^2)\;\log\lambda - \frac{3}{4}\kappa^2\xi^{-1} + \frac{11}{6} + \frac{16}{3}\kappa + \frac{23}{12}\kappa^2 \quad (2.22)$$

$$\widehat{\Pi}^{00}(\lambda) = \frac{1}{4}\kappa^2\xi^{-1}\lambda^{-1} + (\frac{3}{2}\kappa^2\xi^{-1} - \kappa)\log\lambda + \frac{3}{2}\kappa^2\xi^{-1} - \frac{8}{3}\kappa \quad (2.23)$$

So for all small ξ the Lee and Yang theory predicts that the derivative of the vacuum polarization on the light cone shall be

$$\Pi'(0) = \frac{e^2}{48\pi^2 m^2}\left(\frac{3}{4}\kappa^2\xi^{-1} + (1+3\kappa-\frac{1}{2}\kappa^2)\log\xi^{-1} - \frac{16}{15} - \frac{16}{3}\kappa - \frac{37}{12}\kappa^2\right) \quad (2.24)$$

3 Conclusions

It behoves us to first contrast this calculation with that undertaken by Beg[(7)]. Beg cut off the dispersion integral at the thresh-hold where the metric in Hilbert space ceases to be positive definite for meson states of zero total charge, $k^2 = -(M_0+M_1)^2$: i.e., he wrote, without adducing any justification,

$$\Pi'(0)_{BEG} = -1\Pi \int_{4m^2}^{M+M_1)^2} \frac{dS}{s^2}\,\text{Im}\,\Pi(-S) \quad (3.1)$$

Thence

$$\Pi'(0)_{BEQ} = \frac{e^2}{48\pi^2 m^2} x \frac{1}{4}\kappa^2\xi^{-1} \;+\; \text{less singular terms},\ \kappa \neq 0$$

$$= \frac{e^2}{48\pi^2 m^2} x \frac{1}{4}\kappa^2\xi^{-1} \;+\; \text{finite terms},\ \kappa = 0 \tag{3.2}$$

If one accepts the conjecture by Lee[3] that the most singular parts of higher order graphs will serve to cancel the divergences in lower order, it follows that for $\kappa = 0$ an additive factor of the form $\log(\xi/\alpha)$ will appear to cancel the log ξ^{-1} divergence. Hence for $\kappa = 0$ one gets the explicit expression

$$\Pi'(0) = \frac{1}{m^2}\frac{\alpha\log\alpha}{12\pi} + \frac{1}{m^2}O(\alpha) \tag{3.3}$$

where the fine structure constant $\alpha = \frac{e^2}{4\pi}$. However for $\kappa \neq 0$ this conjecture fails to lead to any definite numerical prediction.

We refer the reader to Beg's paper [7] for a short disussion of possible experimental measurement of the vacuum polarization.

We conclude by presenting an interpretive criticim of the Lee and Yang theory of charged spin one. The field equations deducible from the Lagrangian density utilised in this theory are

$$\xi D_\nu D_\rho \phi_\rho + D_\mu G_{\mu\nu} - m^2\phi_\nu + ie\kappa\phi_\mu F_{\mu\nu} = 0 \tag{3.4}$$

where

$$G_{\mu\nu} = D_\mu\phi_\nu - D_\nu\phi_\mu \;\; ; \;\; D_\mu = \partial_\mu - ieA_\mu \tag{3.5}$$

contracting with D_ν and using the relation

$$D_\mu D_\nu - D_\nu D_\mu = -ieF_{\mu\nu} \tag{3.6}$$

gives

$$\xi D^2 D_\rho\phi_\rho - m^2 D_\nu\phi_\nu = -\frac{ie}{2}(\kappa - 1)F_{\mu\nu}G_{\mu\nu} \tag{3.7}$$

We write

$$\phi_\mu = \phi^1_\mu + \phi^0_\mu \tag{3.8}$$

defining

$$m^2\phi^0_\mu = \xi D_\mu D_\rho\phi_\rho \tag{3.9}$$

and setting

$$\kappa = 1 \tag{3.10}$$

Then

$$D_\mu D_\nu\phi^0_\nu - M_0^2\phi^0_\mu = 0 \tag{3.11}$$

Defining

$$G^1_{\mu\nu} = D_\mu\phi^1_\nu - D_\nu\Phi^1_\mu \qquad 3.12$$

we find that

$$D_\mu G^1_{\mu\nu} - m^2\phi^1_\nu + ieF_{\mu\nu}\phi^1_\mu = 0 \tag{3.13}$$

This equation implies

$$D_\mu\phi^1_\mu \tag{3.14}$$

The identification of ϕ^0_μ as the spin zero part of ϕ_μ, and ϕ^1_μ as the spin one part is thus unambiguous for $\kappa = +1$.[8] It is also seen that for $\kappa = +1$ the c-number Lagrangian density describes a decoupled system wherein there is no direct interaction of the form

$$\text{(spin one)} \ \to \text{(photon)} + (\text{ spin zero}) \tag{3.15}$$

It follows that were the second quantized theory to properly reproduce this qualitative feature of the c-number theory for $\kappa = 1$, then one should find that the contribution to Im $\Pi(k^2)$ associated with the process (3.15) should vanish to order e^2, though perhaps it could be non-zero to higher order of e as a consequence of some higher order effect. However an inspection of Table I shows that

$$\mathrm{Tr}^{01} \neq 0 \ \ , \ \ \kappa = 1 \tag{3.16}$$

so that the second quantised theory fails to correspond in a simple manner with the c-number theory. On the other hand, in the usual Q.E.D. a parallel situation occurs in the scattering of light by light: this process does not occur in vacuo in the c-number theory , and so appears in the q-number theory only in orders ≥ 4; i.e. in the theory of charged spin $\frac{1}{2}$ the c-number and the q-number theories maintain the same qualitative features in lower order calculations. We therefore regard this defect of the Lee and Yang theory as being quite serious: in another paper [12] we attempt to devise a theory of spin one by paying less rigorous attention to the canonical formalism but giving more regard to this principle of correspondence.

Appendix I

In this Appendix we develop a novel matrix method for dealing with tensors. The method relies on the well known properties of the Kemmer [9] algebra:

$$\beta_{\mu\rho\nu} + \beta_{\nu\rho\mu} = \beta_\mu\delta_{\rho\nu} + \beta_\nu\delta_{\rho\mu} \qquad A1.1)$$

A particular 10x10 irreducible representation of the algebra is given by Roman[10]. In this representation it is easy to demonstrate that the products $\beta_{\mu\nu} = \beta_\mu\beta_\nu$ are of the form of the direct sum of a six 6x6 matrix and a 4x4 marix, the corresponding projection operators for the disjoint spaces being (3-M) and (M-2) respectively, where[11]

$$M = \beta_\mu\beta_\mu \tag{A1.2}$$

It follows from A1.1 that

$$(3 - M)\beta_\mu = \beta_\mu(M - 2) \tag{A1.3}$$

We call the 4x4 part of $\beta_{\mu\nu}$ $\hat{\beta}_{\mu\nu}$ so that

$$(M-2)\beta_{\mu\nu} = O_{4x4}\widehat{\beta}_{\mu\nu} \quad \text{(direct sum)} \tag{AI.4}$$

$\widehat{\beta}_{\mu\nu}$ has components

$$[\widehat{\beta}_{\mu\nu}]_{\beta\alpha} = \delta_{\mu\nu}\beta_{\beta\alpha} - \delta_{\mu\alpha}\delta_{\nu\beta} \tag{AI.5}$$

It is also convenient to define the 10x10 matrix

$$H_{\mu\nu} = \beta_{\nu\mu} - \delta_{\mu\nu} \tag{AI.7}$$

so that the corrsponding 4x4 matrix has components

$$[\widehat{H}_{\mu\nu}]_{\beta\alpha} = -\delta_{\alpha\beta}\delta_{\nu\alpha} \tag{AI.7}$$

The special virtue of this unusual procedure is that when we calculate the traces which occur in the calculation of vacuum polarization we shall be evaluating traces like

$$\mathrm{Tr}\widehat{\beta}_{\mu\nu}\widehat{\beta}_{\rho\sigma}\widehat{H}_{\gamma\delta} = \mathrm{Tr}(M-2)\beta_{\mu\nu}\beta_{\rho\sigma}H_{\gamma\delta} = \mathrm{Tr}(3 - M0\beta_{\nu\rho\sigma}H_{\gamma\delta}\beta_{\mu} \tag{AI.8}$$

These expressions may be readily reduced by using the the well-known properties of the usual Kemmer matrices.

For completeness we note the following traces deducible directly from (A1.7):

(A-I.9)
$$\begin{cases} \mathrm{Tr}\,\overline{H}_{\mu\nu} = -\,\delta_{\mu\nu}\,, \\ \mathrm{Tr}\,\overline{H}_{\mu\nu}\overline{H}_{\varrho\sigma} = \delta_{\nu\varrho}\delta_{\sigma\mu}\,, \\ \mathrm{Tr}\,\overline{H}_{\mu\nu}\overline{H}_{\varrho\sigma}\overline{H}_{\alpha\beta} = -\,\delta_{\nu\varrho}\delta_{\sigma\alpha}\delta_{\beta\mu}\,. \end{cases}$$

The attentive reader will see that the trace of an arbitrary number of ordinary β matrices may be readily found by first determining the trace of products $\widehat{H}_{\mu\nu}$, then of $\widehat{\beta}_{\mu\nu}$.

(A-I.9)
$$\begin{cases} \mathrm{Tr}\,\overline{H}_{\mu\nu} = -\,\delta_{\mu\nu}\,, \\ \mathrm{Tr}\,\overline{H}_{\mu\nu}\overline{H}_{\varrho\sigma} = \delta_{\nu\varrho}\delta_{\sigma\mu}\,, \\ \mathrm{Tr}\,\overline{H}_{\mu\nu}\overline{H}_{\varrho\sigma}\overline{H}_{\alpha\beta} = -\,\delta_{\nu\varrho}\delta_{\sigma\alpha}\delta_{\beta\mu}\,. \end{cases}$$

Appendix II

The purpose of this Appendix is to sketch the calculation of the traces Tr^{ab} utilizing the matrix formalism developed in the preceding Appendix. In terms of this notation the numerator terms in the propagators are the 4×4 matrices

(A-II.1) $$N^1(p) = m^{-2}[(\bar{\beta}p)^2 - (p^2 + m^2)]\,,$$

(A-II.2) $$N^0(p) = -\,m^{-2}[(\bar{\beta}p)^2 - p^2]\,,$$

while the 3-vertex is the 4×4 matrix

$$V_\mu(p''p') = i(1+\varkappa)(\bar\beta_\mu\bar\beta p' + \bar\beta p''\bar\beta_\mu) - \\ - i(\varkappa+\xi)(\bar\beta_\mu\bar\beta p'' + \bar\beta p'\beta_\mu) + i\xi(p'+p'')_\mu \,. \tag{A-II.3}$$

Rather than evaluate the sought traces directly, much labour is saved by first evaluating the following traces:

$$\left\{\begin{aligned} \mathrm{Sp}^{00} &= \mathrm{Tr}\{[(\bar\beta p')^2 - p'^2]\, V_\mu(p', p'')[(\bar\beta p'')^2 - p''^2]\, V_\mu(p'', p')\}\,, \\ \mathrm{Sp}^{+0} &= \mathrm{Tr}\{p'^2\, V_\mu(p', p'')[(\bar\beta p'')^2 - p''^2]\, V_\mu(p'', p')\}\,, \\ \mathrm{Sp}^{0+} &= \mathrm{Tr}\{[(\bar\beta p')^2 - p'^2]\, V_\mu(p', p'')\, p''^2\, V_\mu(p'', p')\}\,, \\ \mathrm{Sp}^{++} &= \mathrm{Tr}\{p'^2\, V_\mu(p', p'')\, p''^2\, V_\mu(p'', p')\}\,, \\ \mathrm{Sp}^{11} &= \mathrm{Tr}\{(\bar\beta p')^2\, V_\mu(p', p'')(\bar\beta p'')^2\, V_\mu(p'', p')\}\,, \\ \mathrm{Sp}^{10} &= \mathrm{Tr}\{(\bar\beta p')^2\, V_\mu(p', p'')[(\bar\beta p'')^2 - p''^2]\, V_\mu(p'', p')\}\,. \end{aligned}\right. \tag{A-II.4}$$

The above traces are evaluated subject to the kinematic constraint

$$k = p' - p''\,. \tag{A-II.5}$$

Noting that Sp^{0+} is readily found from Sp^{+0}, we have explicitly calculated Sp^{00}. Sp^{-1}, and Sp^{00}, thus determining one more trace than is *sufficient* for our needs in order to be able to carry out a consistency check, using the relation

$$\mathrm{Sp}^{00} + \mathrm{Sp}^{+0} + \mathrm{Sp}^{0+} + \mathrm{Sp}^{++} = \mathrm{Sp}^{11}\,. \tag{A-II.6}$$

Finally Sp^{10} is found from the relation

$$\mathrm{Sp}^{10} = \mathrm{Sp}^{00} + \mathrm{Sp}^{+0}. \tag{A-II.7}$$

These six traces are set down in Table I. As well as being utilized in this calculation, their generality enables them to be used in a somewhat different calculation (9). The actual traces sought are found by making the trivial substitutions

$$\left\{\begin{aligned} \mathrm{Tr}^{11}(k^2) &= m^{-6}\,\mathrm{Sp}^{11}[p'^2 = p''^2 = M_1^2]\,, \\ \mathrm{Tr}^{10}(k^2) &= -\,m^{-6}\,\mathrm{Sp}^{10}[p'^2 = M_1;\ p''^2 = M_0^2]\,, \\ \mathrm{Tr}^{00}(k^2) &= m^{-6}\,\mathrm{Sp}^{00}[p'^2 = p''^2 = M_0^2]\,, \end{aligned}\right. \tag{A-II.8}$$

whilst

$$8\,\mathrm{Tr}^{01} = \mathrm{Tr}^{10}\,. \tag{A-II.9}$$

Tr^{11} and Tr^{00} are written down in Sect. 2. Tr^{01} has the value

$$\mathrm{Tr}^{01}(m^2x) = c_3x^3 + c_2x^2 + c_1x + c_0\,, \tag{A-II.10}$$

where

$$\begin{cases} c_3 = -\frac{1}{4}\varkappa^2\,, \\ c_2 = \frac{1}{2}(-\varkappa^2\xi^{-1} + 1 + 4\varkappa)\,, \\ c_1 = \frac{1}{4}(-\varkappa^2\xi^{-1} + 4\xi^{-1} + 4\varkappa\xi^{-1} - 6\varkappa^2\xi^{-1} - 8 + 4\varkappa + 3\varkappa^2)\,, \\ c_0 = \frac{1}{2}(\xi^{-2} - 2\varkappa\xi^{-1} + \varkappa^2\xi^{-2} - 2\xi^{-1} + 4\varkappa\xi^{-1} - 2\varkappa\xi^{-1} + 1 - 2\varkappa + \varkappa^2)\,. \end{cases} \tag{A-II.11}$$

TABLE I. – *Table showing the coefficients of the terms in the traces* Sp^{ab} *defined in Appendix* II. Blank entries denote zero. For compactness, the number of columns has been reduced by « doubling » four columns. The arrangement is illustrated by the following example: for Sp^{+0} the coefficient of $k^4p'^2/2$ is $1+3\varkappa+\varkappa^2-\varkappa\xi$, while the coefficient of $k^4p''^2/2$ is zero.

Trace	Term					
	$k^6/4$	$k^4p'^2/2$	$k^2p'^2(p'^2+p''^2)/4$	$k^2p'^2p''^2$	$p'^2(p'^2+p''^2)^2/4$	$p'^4p''^2$
		$k^4p''^2/2$	$k^2p''^2(p'^2+p''^2)/4$		$p''^2(p'^2+p''^2)^2/4$	$p'^2p''^4$
Sp^{00}	$\varkappa^2$	$-\varkappa^2+\varkappa\xi$	$\varkappa^2-4\varkappa\xi$	$-\varkappa^2\quad+\xi^2$	$2\varkappa\xi$	$-2\varkappa\xi-2\xi^2$
		$-\varkappa^2+\varkappa\xi$	$\varkappa^2-4\varkappa\xi$		$2\varkappa\xi$	$-2\varkappa\xi-2\xi^2$
Sp^{+0}		$1+3\varkappa+\varkappa^2-\varkappa\xi$	$-4-12\varkappa-4\varkappa^2+4\varkappa\xi$	$3\varkappa+3\varkappa^2+3\xi+3\varkappa\xi-\xi^2$	$2+6\varkappa+2\varkappa^2-2\varkappa\xi$	$1-3\varkappa-2\varkappa^2-3\xi-\varkappa\xi+2\xi^2$
						$-3\varkappa\quad-3\xi+3\varkappa\xi+5\xi^2$
Sp^{0+}				$3\varkappa+3\varkappa^2+3\xi+3\varkappa\xi-\varkappa^2$		$-3\varkappa\quad-3\xi+3\varkappa\xi+5\xi^2$
		$1+3\varkappa+\varkappa^2-\varkappa\xi$	$-4-12\varkappa-4\varkappa^2+4\varkappa\xi$		$2+6\varkappa+2\varkappa^2-2\varkappa\xi$	$1-3\varkappa-2\varkappa^2-3\xi-x\xi+2\xi^2$
Sp^{++}				$3-6\varkappa-6\varkappa^2-6\xi-6\varkappa\xi+\xi^2$	-9	$-9\quad+6\xi\quad-5\xi^2$
Sp^{11}	$\varkappa^2$	$1+3\varkappa$	$-4-12\varkappa-3\varkappa^2$	$3\quad-\varkappa^2$	$2+6\varkappa+2\varkappa^2$	$-8-6\varkappa-2\varkappa^2$
		$1+3\varkappa$	$-4-12\varkappa-3\varkappa^2$		$2+6\varkappa+2\varkappa^2$	$-8-6\varkappa-2\varkappa^2$
Sp^{10}	$\varkappa^2$	$1+3\varkappa$	$-4-12\varkappa-3\varkappa^2$	$3\varkappa+2\varkappa^2+3\xi+3\varkappa\xi$	$2+6\varkappa+2\varkappa^2$	$1-3\varkappa-2\varkappa^2-3\xi-3\varkappa\xi$
		$-\varkappa^2+\varkappa\xi$	$\varkappa^2-4\varkappa\xi$		$2\varkappa\xi$	$-3\varkappa\quad-3\xi+\varkappa\xi+3\xi^2$

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