OZNAKI: A NEW MEDIUM FOR MATHEMATICIANS

- MINI-ROBOTS FOR MINI-MATHEMATICIANS -

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The "I was never any good at Maths" syndrome is only too familiar. Soon to spearhead an attack on its causes will be what sounds a rather fearsome platoon - ZONKY, OZGRA, WHAM, CROC and PLUSMINUS - the vanguard of the OZNAKI army.

THE OZNAKI PROJECT

OZNAKI, the Polish word for SYMBOLS, is the name of a research project being conducted at La Trobe University in which computer technology is harnessed in the design of a mathematical learning environment. The aim has *been* to provide concrete material that interacts rather than reacts with the child. The environment is to be one in which the student works, plays, experiments, orates and develops his own mathematical creations.

Now everyone knows that real mathematicians play with symbols. However these symbols are tz the mathematician far more than doodles on paper; they represent mathematical ideas. How can a child meaningfully manipulate symbols if he has not reached the formal operations stage? But on the same basis a Martian might wonder how a young child could use the abstract symbolism contained in a "natural" language such as English. What computer technology offers is the possibility of transforming mathematical symbols into commands that cause obvious effects, from audible clangs *to* TV screen flasks.

In the OZNAKI Project we provide a symbolic language in which the symbols introduced descrt3e (encode) concrete operations that are in very close structural correspondence to the mathematical operation or relation. In mathematics jargon this correspondence is called an ISOMORPHISM. That is, in OZNAKI we utilise computer technology to provide concrete embodiments for the mathematical concepts that are symbolised,

CHILDREN AS MATHEMATICIANS

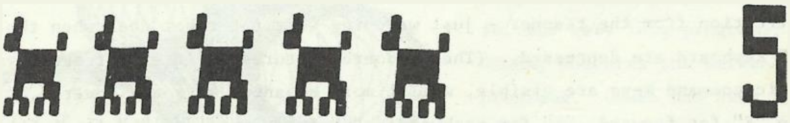
The OZNAKI Project has been very strongly motivated by the philosophy expressed so well by Seymour **Papert** that children should do mathematics rather than **lea=** mathematics. To this end Papert and Feurzeig developed LOGO, which gave a first inkling that computer technology had something significantly new to **offer** teachers of mathematics. The LOGO experiment **never** really yielded a practical school system due to **its** narrow focus and the high cost of computers until very recently.

The idea that a computer should offer a user a **great** range of possible experiences of a creative flavour was first recognised by Alan Kay and Deidre Goldberg of the XEROX Corporation, who developed the fascinating language SMALLTALK.

OZNAKI also grew out of this author's preoccupation with the cognitive aspects of problem solving by mathematicians and physicists.

THE PLUSMINUS

The youngest child who plays with the PLUSMINUS is confronted by two large keys, marked + and -, and a TV screen, on which a large "0" is displayed. If the large + key is struck, there is a sharp metallic click, a train appears on the screen, and the number displayed becomes a 1. Each further time the + key is struck, one further train appears, and the new total of trains is displayed. If the - key is struck, there is a loud click, the train on the end of the row disappears, and the number display is altered to show the new total. In place of trains, birds or dogs may be displayed.



Now clearly in PLUSMINUS the number display together with the row of screen animals provide the same sort of embodiment of numbers as is given by the more traditional concrete materials. The PLUSMINUS associates a digit with a definite number of objects, and with the number of times the + key has *been* struck in counting up to the screen total (or counting down to make the TV animals disappear). But unlike rods, the PLUSMINUS also provides a concrete representation for the symbols + and -; the actual OPERATIONS of addition/subtraction are modelled. Traditional materials can
only show the before and after pictures for an arithmetical operation, and cannot model the operation itself.

A PLUSMINUS THAT UNDERSTANDS SPEECH

The original PLUSMINUS constructed for the OZNAKI project featured a row of 9 bright lights, an electronic number display, and two very large keys marked + and -. (There is a picture of this calculator type PLUSMINUS operated by a 2 year-old in the authors article OZNAKI in "SET TWO", 1976). The development of the OZNAKI microcomputer, our WIZARD'S BOX, led to PLUSMINUS becoming a program controlled by the microcomputer. Because of the incredible flexibility of computer control, PLUS-MINUS has far greater capabilities than those described so far, and further capabilities could be added at the teacher's discretion. Thus providing the child with a larger keyboard, one can enable the child to select the type of screen animal, while striking a key displaying a digit will make the appropriate number of TV animals (or trains) appear. The possibilities if a larger student keyboard included a multiplication key are obvious.

Recently computer technology has been applied to the needs of the blind in producing braille calculators that "speak" the digits struck and the result of calculations. The braille calculators are examples of "Speech Output" devices. The OZNAKI Project has sought to study the educational implications of the use of "Speech Input" for our PLUSMINUS. This work represents an entirely new dimension in educational technology.

Using surprisingly simple hardware in conjunction with software developed by us it is possible for the PLUSMINUS, and other OZNAKI systems, to accept speech input. After a training session in which a child talks into a microphone attached to the microcomputer, while the teacher keys into the keyboard the corresponding written words, the words spoken by the child are recognised by the microcomputer. In the Speech PLUSMINUS the spoken words "plus one" function exactly like the + keystroke, and the words "minus one" like the - key. The words "six" "dogs" will cause just 6 dogs to appear on the TV.

ZONKY

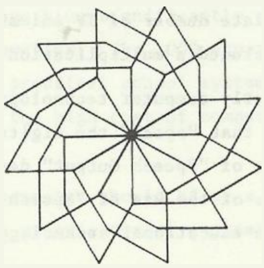
Now, what is ZONKY? In the first place ZONKY is a robot that crawls about the floor or tabletop connected by just 3 thin wires to the microcomputer Wizard Box. On ZONKY are marked the FORWARD direction, and the directions for RIGHT and LEFT turns (on the spot) which take place about a central point where a felt tipped pen is mounted. This pen, if depressed, marks out the robot's trail. How does a child direct BONNY? The method is simple and the child unconsciously learns basic mathematical ideas in a fun atmosphere. At his first lesson the pupil learns to give ZONKY the basic commands with a minimum of instruction from the teacher - just watching what the robot does when the various keys on the special OZ keyboard are depressed. (The keyboard features a variety of overlays so that for a beginner the basic command keys are visible, whilst more advanced keys are covered.) For ZONKY the basic commands are "F" for forward, "B" for backwards, "R" for right, and "L" for left. While performing these movements the automaton can also be ordered to honk "H", (a) light its lights "A".

"J" is the command for ZONKY to "sing" the first bar of the tune "Jingle Bells". A string of commands - like an English command - must be terminated by an exclamation mark ":". (There is a version of the command language for pre-schoolers where the ":" is not required, and only one command at a time can be performed.) To illustrate, following the command 3AF2HR: the robot makes 3 steps forward with its lights flashing, then makes 2 turns right (clockwise) on the spot whilst honking its horn.

The pre-schooler who first encounters PLUSMINUS is limited to the use of just two symbols, "+" and "-", and cannot invent any of his own. This remains true even for the extended PLUSMINUS, where always each key has a fixed meaning. However children directing ZONKY, and other OZNAKI systems actually name ("define") new symbols. This may sound extraordinary and difficult, but in the OZNAKI framework it is elementary. There is one very special command on the keyboard, which is marked by the letter "Z". When this key is depressed, the following list of commands is to be REMEMBERED, all of them up to the exclamation mark ":". And what is to be remembered can have a name chosen from "V", "W", "X", or "Y".

To return to a child directing ZONKY as above by the string of commands 3AF2HR: ; this string of commands could be called X by punching in ZX3AF2HR: Then on punching in the command 2X the sequence of commands whose name is X could be performed twice. As ZONKY can mark his (or its:) trail, the remember feature enables a symbol to be associated not only with a string of commands, but also with a geometrical figure. Thus combining parts of a picture to get a whole is clearly related to stringing commands together.

Another example of ZONKY maths is in order. The command C causes ZONKY to emit a Clang sound. Likewise, the command 3C: will give three distinct clangs. Suppose a child punches in ZX3C: so that the command X will stand for "Three clangs <do it Then if the student, (or his teacher) punches in 2X1 he will hear six clangs and his teacher would ask

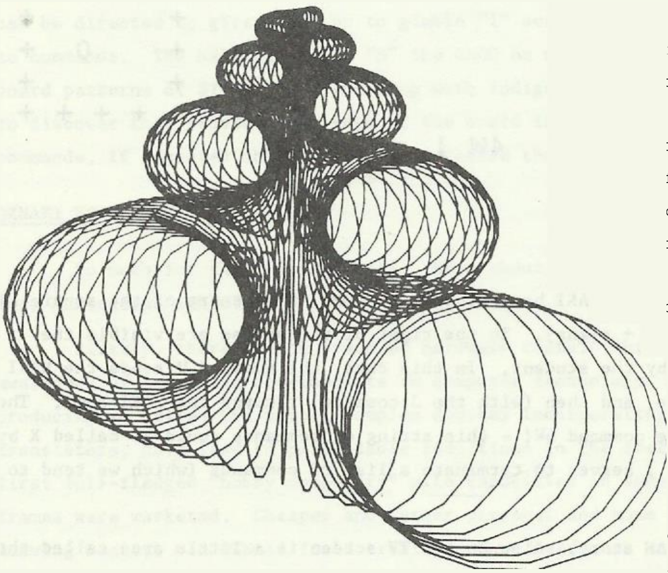


how many X commands were performed. Thus ZONKY gives a very purposeful introduction to algebra to primary children.

OZGRA

OZGRA, also known as OZ-Graphics, is a computer language which enables the user to produce an astonishing variety of line drawings/patterns on a graphic screen using simple commands. On the screen is an obedient NAKI , visible as a flashing dot, which can leave a track on the screen as it moves. This NAKI obeys the same moving commands as ZONKY, except that the R command is for a (clockwise) turn on the spot of only one degree.

In OZGRA one can do everything in the way of line-graphics that can be done in LOGO. In fact OZGRA has some



ipulation primitives not available in LOGO. ms consist of separate lines each with a line : as in the BASIC programming language. This LOGO is useful for very long programs, but, on to OZGRA, obscures the mathematical f the programs. OZGRA has been tailored to the capabilities of a microcomputer with limited ike LOGO. Unfortunately, like LOGO, OZGRA fine graphics screen which costs of the order of two thousand dollars.

Thus OZGRA, although cheaper than LOGO, is far too expensive for regular school use. This is a great pity, in that Papert and his co-workers at M.I.T. elaborated a rather beautiful "Turtle. Geometry" based on LOGO, which is equally applicable to OZGRA.

WHAM

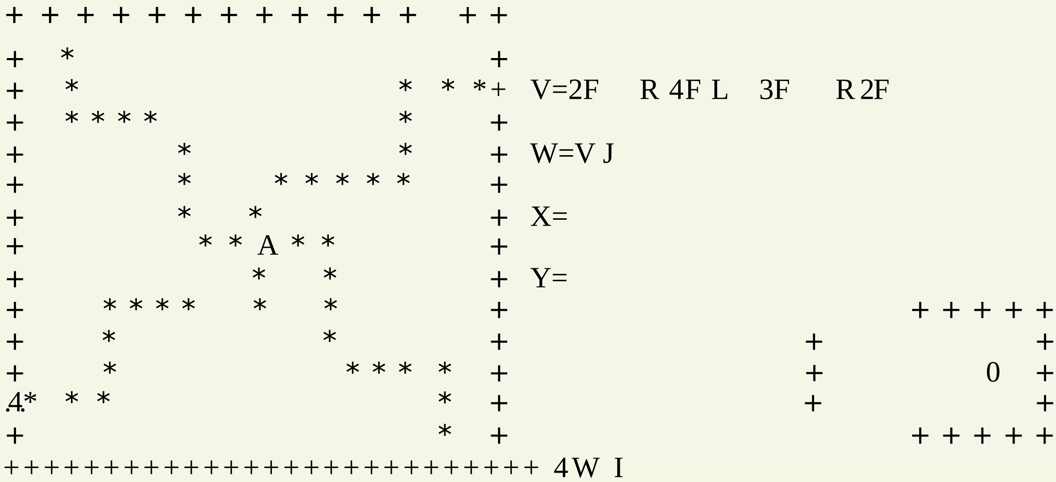
In WHAM the student controls with a typewriter style keyboard the happenings on a TV screen. Most of the screen is taken up with a square area in which a strange creature called a NAZI lives. This creature obeys the same basic commands as the OZNAKI robot ZONKY, namely FORWARD, BACK, RIGHT, and LEFT. These commands are simply represented by the typewriter keys F, B, R, and L. However the NAZI only ever turns through ninety degrees, e.g. from being

V

facing down (or South) on the TV screen, to the state

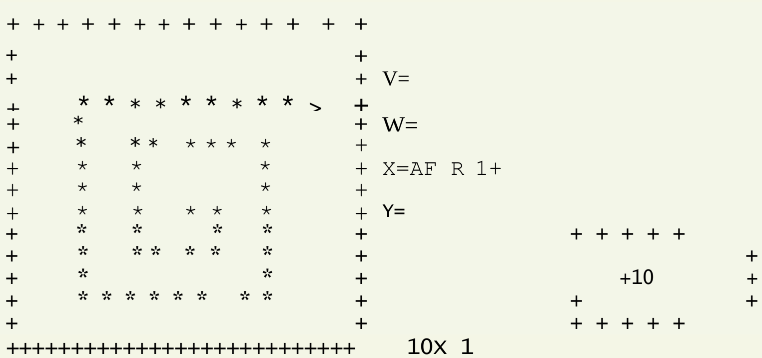
after the command R, when the NAZI heading is to the West.

As the NAKI roams his territory he leaves 'a trail of asterisk filled squares, and so forms a design, which can also be printed out.



In this copy of the WHAM screen, the NAKI has returned to HOME, the centre of the square NAKI territory (which has a fence composed of + signs). To the right of the screen are visible the command streams (i.e. programs) named by the student. In this case, the command V sends the NAKI on a path drawing one arm of the figure, and then (with the J command) returns the NAKI home. The entire logo shown can be drawn with the command 4W: - this string of commands could be called X by punching in ZX4W: where the imperative : serves to terminate a list of commands (which we tend to call a command stream).

But this is only part of the WHAM story. Also on the TV screen is a little area called the ACCUMULATOR, in which there is a number. The ACCUMULATOR is the number display for an electronic calculator. The usual number operators +,-,/ (for divide) and * (for multiply) are on the typewriter keyboard. The WHAM calculator has reverse Polish syntax (like Hewlett-Packard calculators) the E key standing for the number command ENTER. Thus to multiply 2 by 3, one would press in sequence the keys 2E3*.'. The answer 6 will be computed and appear in the ACCUMULATOR. The WHAM calculator accepts not only specific numbers, but also uses A *as* the name of the number currently in the ACCUMULATOR. Thus to square the number in the ACCUMULATOR one can type in the command stream A*1. Now just as mathematics is used to guide a rocket to the moon, so the student himself can use very simple algebra in WHAM to direct the NAKI to draw spectacular patterns.



In the command stream ZXAFR1+1 typed in by the student, the command X stands for the stream of commands AFR1+. The NAKI set out from the middle of the screen, when A was zero. For each X command, the NAKI drew one further side of the "squiral' and the number in the ACCUMULATOR increased by one. The creation of such a striking logo after a modest amount of mathematical labour is a reward for the student of real impact, hence the name WHAM.

CROC

CROC may be regarded as a version of WHAM played on a hexagonal board. In fact there are two CROC boards, a simple hexagonal pattern on the TV screen, and a rather beautiful "expanded" version of the CROC board on a large sheet. On the paper CROC board a pattern of interlaced crocodiles is used, the motif based on the lizards in Maurits Escher's "Reptile" drawing. In CROC a NAKI can be directed to gYre, "Y", or to glmble "I" over the board going from CROC to CROC in accordance to commands. The NAKI can Sting "S" the CROC on which it currently sits, so that moving about the board patterns of STUNG CROCS (burning with indignation) can be seen. It is somewhat astonishing to discover that provided the edge of the board is not encountered, any programmed sequence of commands, if repeated three times, will return the NAKI to its initial CROC.

OZNAKI FOR AUSTRALIAN SCHOOLS

So much for the driving ideas. What about the practical aspects? How can OZNAKI be fitted into Australian schools?

Firstly, there is the issue of hardware costs. This project was conceived in 1975 with an awareness of impending developments in computer technology. Improvements in the technology of producing so called LSI chips, complex devices incorporating the equivalent of many thousands of transistors, have lead to predictable reductions in the cost of computer power. Late in 1976 the first full-fledged "hobby computers" with capacities in some ways superior to the traditional main-frames were marketed. Cheaper and better personal and home (1) computers are currently on the drawing boards. In OZNAKI field trials we have used such hobby computers to provide the necessary computer control for our systems. For program storage we have used both acoustic tapes and also the more convenient "floppy discs". Thus commercially available microcomputers together with our inexpensive robot, and special keyboards, are the basic hardware requirements. At a cost in 1977 of about $1000, but predictably cheaper in 1978, and much cheaper in 1983.

The second practical problem in developing OZNAKI is in writing of teacher's guides and primers. Primers that describe the capabilities of the various existing OZNAKI systems have been written, but these require teacher initiative to be translated into lessons. We are aiming to develop a number of teaching modules comprising a structured series of lessons and projects, with notes for teacher use. One such module, the Projection Module, aiming at developing spatial concepts, is described below.

EVALUATION

How does one evaluate the educational robotics developed by the OZNAKI Project? Essentially there —e t,..70 (intersecting) forms of evaluation. Typically, educational systems are evaluated on a behaviourist basis: in what way has the student's behaviour, (class marks and the like) been altered by his experience. Of special interest to OZNAKI is the evaluation in terms of cognitive structures. Both are important and valid aspects of evaluation.

ZONKY provides a "multiple-embodiment" of number concepts as numbers relate to lengths, to repeated control characters appearing on the TV, to accompanying sounds and to time durations. A study showing the effects of the OZNAKI experience on various quantity operations would be useful: this aspect of OZNAKI follows the pattern established by Zoltan Dienes. It is, however, of special importance to evaluate those many features of OZNAKI not available in any previous materials.

Evaluation of PLUSMINUS

We plan to focus on the development of cognitive structures in a planned study of the PLUSMINUS. The PLUSMINUS provides concrete modelling for the operations of addition and subtraction. A more detailed understanding of the significance of the PLUSMINUS can be derived from Gelman's studies of the development of early number concepts. To quote Gelman: "The cognitive processes by which people determine some quantity, such as the numerosity of a set of objects, are called ESTIMATORS. The cognitive processes by which people determine the consequences of transforming a quantity in various ways are called OPERATORS." As Gelman asserts, "operators are more central to a mature conception of number". It appears from various evidence, including the studies of Sinclair and Inhelder at Geneva, that the child's "number scheme is a central quantity scheme which facilitates the development of other quantity concepts". It is in fact rather difficult to test young children's number concepts, but Gelman developed a technique. In her experiments preschoolers were trained to anticipate a certain arithmetical relationship between two collections of objects. Then, using "magic", the interviewer surreptitiously altered one collection. The child's reaction of surprise gave a non-verbal measure. We plan to follow the path pioneered by Gelman and measure the relative enhancement of number concepts in preschoolers who have played with the PLUSMINUS, the Speech-Input PLUSMINUS, and other materials.

THE PROJECTION TEACHING MODULE

ZONKY and WHAM involve NAKIs that can move backwards and forwards, and turn on the spot - just as people can. All the commands to the NAKIs relate to the NAKIs current heading - not to the reference frame of the student. So that in order to control these NAKIs the child has to imagine that he is the NAKI: the student is called upon to develop skills at what Piaget terms projection.

For the younger child, space relates to himself as focus (is egocentric), and topological relationships but not metrical relationships are recognised. The mature adult view of space is one in which a person views himself and other objects in relation to the fixed Cartesian frame of the Earth. In seeking to understand how the child's views of space develop, Piaget's comments are of special significance. "... spatial concepts are internalised actions and not merely mental images of external things or events - or even the images of the results of actions". That is, children learn the mental manipulation of spatial concepts through their own movements. Piaget identified Projection as the link between the topological egocentric view of the young child and the Euclidean view of formal reasoners. Thus the NAKIs provide a unique tool in developing spatial concepts in children at the stage of concrete operations.

Dr. David Green and the writer have developed a teaching module for teaching projection via OZNAKI. The module comprises just ten lessons. It has been taught with up to six children using one particular microcomputer Wizard Box. In the first lesson children encounter ZONKY, our "real" robot, then they progress to WHAM, while the final lesson is on OZNAKI "LIFE". In dealing with ZONKY, and in later WHAM sessions, the students are encouraged to act out the NAKIs planned moves. Included in the lessons are the tasks of guiding ZONKY past obstacles, and a similar, but more demanding task with the WHAM NAKI. The "LIFE" lesson involves a NAKI which obeys the commands N for North (Up), S for South (Down), E for East (Right), W for West (Left) - the same as used in map reading. The module also includes an introduction to the algorithmic aspects of OZNAKI including the calculator maths in WHAM.

EVALUATION OF THE PROJECTION MODULE

At the date of writing this article in late 1977 an evaluation study of the Projection Module was underway. This study has the following phases:

Pretesting of all students involved.
Selection of Experimental and Control Groups.
Teaching the Projection Module to the E Group.
Post-testing of both E and C Groups.

The students participating in the study are aged 8 to 12, and include both primary and secondary students. The pre-tests and post-tests for the primary school students involve a Piagetian type interview, whilst the tests for the secondary students involve a multi-choice questionnaire.

Primary school students included in the study were pre-tested in individual interviews in which two games were played. In the first interview game a model of three mountains was placed before the child. The interviewer produced sketches of various views of these mountains, and asked the child to place a toy man on the model where this view could be seen. (This is a classic Piaget puzzle.) To obtain a performance parameter, the interviewer noted the time taken in each placement. The second game, devised by the writer, involves a model town and toy car. The first task in this game is for the child to drive the car about the streets of the town, while keeping to the right of the centre-line. Other tasks in this game involve combinations of planning and spatial specifications such as guiding the toy car between two places in the model town making left turns only. Our experimental design requires that the same interview games be used as a post-test.

In high schools, to select the students to participate in the course, and a matched control group, we tested the entire classes available using a printed multi-choice questionnaire. (This test included questions suggested by Mr. Greg. Cornish of A.C.E.R.) A similar questionnaire is to be used in the post-tests.

OTHER MEMBERS OF THE FAMILY

In this paper we have described ZONKY, CROC, WHAM, OZGRA, and PLUSMINUS, the original members of the OZNAKI family. From the viewpoint of computer science, these systems are variants of a universal Tiny Robotics Language. In other words, given the specifications for a new NAKI, its control is readily programmed. We propose to extend this Universal Tiny Robotics Language to include conditional programming constructs of the form

T(first list)(second list)

so that if the accumulator is zero the commands in the first list are performed, else the second. We propose to use the OZNAKI framework to develop a number of other mathematical robots. Under way is the OZNAKI sketchpad system, which will provide a flexible way for students to construct cartoon figures like the animals of PLUSMINUS. ,In fact it will be possible for such screen animals to be programmed to wander over the screen.

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MICROCOMPUTER TECHNOLOGY

There is no satisfactory introductory text. Good articles appear in the following magazines:

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"Peoples Computers", published by PCC, Menlo Park, California.

OZNAKI PRIMERS

Teaching Mathematics via OZ-Graphics

A WHAM Primer

A Teacher's Guide to CROC

The OZNAKI Robotics Language OZ

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