International Conference on Modelling & Simulation, Melbourne 1987

Symmetry of Multidimensional filters: implications for coefficients for software and hardware design

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Abstract

The implications of symmetry for the computational complexity of three dimensional discrete convolutions of mask size $(2K+1)(2K+1)(2K+1)$ are studied. It is shown that even axial symmetry reduces the number of multiplications needed in the evaluation by a factor that tends to 8 with increasing K, whereas the combination of axial and diagonal even symmetry reduces the multiplicative complexity by a factor that tends to 48 with increasing K. Explicit computationally convenient forms of the convolution formula that have been reduced by symmetry considerations are derived for the cases of total and partial axial even symmetry, and total axial and diagonal even symmetry. The methods used and the formulas derived have applications to the computational methods and hardware design for convolution evaluation in diverse areas of multidimensional signal processing.

1 Introduction

Rapid progress in the development of inexpensive powerful processors and multiple processor systems with large local memory space has increasingly made accessible to these systems tasks of higher dimensionality.

In this paper we consider a fundamental operation in the context of discrete linear systems of three dimensions, the convolut The extent of application of the convolution operation is extremely diverse [1,..,5] and does not require introduction. However, frequency, phase, delay, amplitude and edge selectivity are some basic properties. These properties are commonly applied to time, frequency and spatial domain processing, and combinations of these. [1]

The convolution of a three dimensional filter h(i,j,k) with an image x(l,m,n) is defined as

$$y(l,m,n) = \sum_{i=-K}^{K}\sum_{j=-K}^{K}\sum_{k=-K}^{K} h(i,j,k)x(l-i,m-j,n-k) \qquad (1)$$

If we define the backward shift operators

$$\begin{aligned} z_1^a x(l,m,n) &= x(l-a,m,n) \\ z_2^a x(l,m,n) &= x(l,m-a,n) \\ z_3^a x(l,m,n) &= x(l,m,n-a) \end{aligned} \qquad (2)$$

we may simplify Eq(1) as follows:

$$y = \sum_{i=-K}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k)\left[z_1^{-i} z_2^{-j} z_3^{-k}\right]x \tag{3}$$

Note that in this equation, and elsewhere in this paper we have omitted the indices (l,m,n).

The number of multiplications needed to compute just one element $y(l,m,n)$ is $(2K+1)^3$. In this paper we consider the reformulation of the convolution sum for even symmetry about axial planes, and also for even symmetry about diagonal planes. This type of degeneracy is common in practical applications. Expressions are derived involving substantial reductions in the number of multiplications required for computation.

2 Axial Symmetry

In this section the $(2K+1)^3$ convolution sums of Eq(1), as rewritten in terms of the backward shift operators of Eq(2), is reduced by taking into account even symmetry in each of the three axial planes.

To determine the effects of symmetry about the axial plane i=0 we separate the above expression into positive, zero and negative components of 'i' to give

$$\begin{aligned} y = \quad & \textstyle\sum_{i=}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k)\left[z_1^{-i} z_2^{-j} z_3^{-k}\right]x \\ & + \textstyle\sum_{j=-K}^{K} \sum_{k=-K}^{K} h(0,j,k)\left[z_2^{-j} z_3^{-k}\right]x \\ & + \textstyle\sum_{i=-K}^{-1} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k)\left[z_1^{-i} z_2^{-j} z_3^{-k}\right]x \end{aligned} \tag{4}$$

The last term above may be rewritten by changing the sign of 'i' in the summation as

$$\sum_{i=1}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k)\left[z_1^{-i} z_2^{-j} z_3^{-k}\right]x \tag{5}$$

However, symmetry of the filter about the i=0 plane implies that

$$h(i,j,k) = h(-i,j,k) \tag{6}$$

Using equations (5) and (6) we may rewrite Eq(3) as

$$\begin{aligned} y = \quad & \textstyle\sum_{j=-K}^{K} \sum_{k=-K}^{K} h(0,j,k)\left[z_2^{-j} z_3^{-k}\right]x \\ & + \textstyle\sum_{i=1}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k)\left[(z_1^{i} + z_1^{-i}) z_2^{-j} z_3^{-k}\right]x \end{aligned} \tag{7}$$

If we now define the central sum operators such that

$$\begin{aligned} [S_1^a]x &= \left[z_1^a + z_1^{-a}\right]x \\ [S_2^a]x &= \left[z_2^a + z_2^{-a}\right]x \\ [S_3^a]x &= \left[z_3^a + z_3^{-a}\right]x \end{aligned} \tag{8}$$

Then Eq(7) becomes

$$\begin{aligned} y = \quad & \textstyle\sum_{j=-K}^{K} \sum_{k=-K}^{K} h(0,j,k)\left[z_2^{-j} z_3^{-k}\right]x \\ & + \textstyle\sum_{i=1}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k)\left[S_1^{i} z_2^{-j} z_3^{-k}\right]x \end{aligned} \tag{9}$$

Comparison of this expression with the initial convolution of Eq(1) reveals that the number of muliplications required to evaluate these equations has been reduced from $(2K+1)^3$ to $(K+1)(2K+1)^2$, by utilizing the redundancy due to symmetry about the single i=0 plane.

To consider the effects of symmetry about a second axial plane j=0, we separate equation (9) into its negative, zero and positive components in 'j' to give

$$y = \sum_{j=1}^{K}\sum_{k=-K}^{K} h(0,j,k)\left[z_2^{-j}z_3^{-k}\right]x$$
$$+\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{k=-K}^{K} h(i,j,k)\left[S_1^i z_2^{-j}z_3^{-k}\right]x$$
$$+\sum_{k=-K}^{K} h(0,0,k)\left[z_3^{-k}\right]x$$
$$+\sum_{i=1}^{K}\sum_{k=-K}^{K} h(i,0,k)\left[S_1^i z_3^{-k}\right]x$$
$$+\sum_{j=-K}^{-1}\sum_{k=-K}^{K} h(0,j,k)\left[z_2^{-j}z_3^{-k}\right]x$$
$$+\sum_{i=1}^{K}\sum_{j=-K}^{-1}\sum_{k=-K}^{K} h(i,j,k)\left[S_1^i z_2^{-j}z_3^{-k}\right]x \quad (10)$$

For even symmetry about the j=0 plane however we have

$$h(i,j,k) = h(i,-j,k) \quad (11)$$

Changing the sign of 'j' in the last two terms of Eq(10) allows the equation to be rewritten as

$$\begin{aligned} y = &\textstyle\sum_{k=-K}^{K} h(0,0,k)\left[z_3^{-k}\right]x \\ &+\textstyle\sum_{j=1}^{K}\sum_{k=-K}^{K} h(0,j,k)\left[(z_2^{j}+z_2^{-j})z_3^{-k}\right]x \\ &+\textstyle\sum_{i=1}^{K}\sum_{k=-K}^{K} h(i,0,k)\left[S_1^i z_3^{-k}\right]x \\ &+\textstyle\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{k=-K}^{K} h(i,j,k)\left[S_1^i(z_2^{j}+z_2^{-j})z_3^{-k}\right]x \end{aligned} \quad (12)$$

Application of the Central Sum Operators of Eqn (8) reduce this equation to

$$\begin{aligned} y = &\textstyle\sum_{k=-K}^{K} h(0,0,k)\left[z_3^{-k}\right]x \\ &+\textstyle\sum_{j=1}^{K}\sum_{k=-K}^{K} h(0,j,k)\left[S_2^{j}z_3^{-k}\right]x \\ &+\textstyle\sum_{i=1}^{K}\sum_{k=-K}^{K} h(i,0,k)\left[S_1^i z_3^{-k}\right]x \\ &+\textstyle\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{k=-K}^{K} h(i,j,k)\left[S_1^i S_2^{j}z_3^{-k}\right]x \end{aligned} \quad (13)$$

Examination of the limits of the above equation shows that the number of multiplications has been reduced to $(2K+1)(K+1)^2$ allowing for symmetry about two axial planes.

We now consider the remaining axial plane k=0 for which even symmetry implies:

$$h(i,j,k) = h(i,j,-k) \quad (14)$$

we may simplify Eq(1) as follows:

$$y = \sum_{i=-K}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k) \left[z_1^{-i} z_2^{-j} z_3^{-k} \right] x \tag{3}$$

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The last term above may be rewritten by changing the sign of 'i' in the summation as

$$\sum_{i=1}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k) \left[z_1^{-i} z_2^{-j} z_3^{-k} \right] x \tag{5}$$

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$$\begin{aligned} y = \quad & \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(0,j,k) \left[z_2^{-j} z_3^{-k} \right] x \\ & + \sum_{i=1}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k) \left[(z_1^{i} + z_1^{-i}) z_2^{-j} z_3^{-k} \right] x \end{aligned} \tag{7}$$

If we now define the central sum operators such that

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Then Eq(7) becomes

$$\begin{aligned} y = \quad & \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(0,j,k) \left[z_2^{-j} z_3^{-k} \right] x \\ & + \sum_{i=1}^{K} \sum_{j=-K}^{K} \sum_{k=-K}^{K} h(i,j,k) \left[S_1^{i} z_2^{-j} z_3^{-k} \right] x \end{aligned} \tag{9}$$

Comparison of this expression with the initial convolution of Eq(1) reveals that the number of muliplications required to evaluate these equations has been reduced from $(2K+1)^3$ to $(K+1)(2K+1)^2$, by utilizing the redundancy due to symmetry about the single i=0 plane.

To consider the effects of symmetry about a second axial plane j=0, we separate equation (9) into its negative, zero and positive components in 'j' to give

$$
\begin{aligned}
y = &\sum_{j=1}^{K}\sum_{k=-K}^{K} h(0,j,k)\,[z_2^{-j}z_3^{-k}]x \\
&+\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{k=-K}^{K} h(i,j,k)\,[S_1^i z_2^{-j}z_3^{-k}]x \\
&+\sum_{k=-K}^{K} h(0,0,k)\,[z_3^{-k}]x \\
&+\sum_{i=1}^{K}\sum_{k=-K}^{K} h(i,0,k)\,[S_1^i z_3^{-k}]x \\
&+\sum_{j=-K}^{-1}\sum_{k=-K}^{K} h(0,j,k)\,[z_2^{-j}z_3^{-k}]x \\
&+\sum_{i=1}^{K}\sum_{j=-K}^{-1}\sum_{k=-K}^{K} h(i,j,k)\,[S_1^i z_2^{-j}z_3^{-k}]x
\end{aligned}
\tag{10}
$$

For even symmetry about the j=0 plane however we have

$$h(i,j,k) = h(i,-j,k) \tag{11}$$

Changing the sign of 'j' in the last two terms of Eq(10) allows the equation to be rewritten as

$$
\begin{aligned}
y = &\textstyle\sum_{k=-K}^{K} h(0,0,k)\,[z_3^{-k}]x \\
&+\textstyle\sum_{j=1}^{K}\sum_{k=-K}^{K} h(0,j,k)\,[(z_2^{j}+z_2^{-j})z_3^{-k}]x \\
&+\textstyle\sum_{i=1}^{K}\sum_{k=-K}^{K} h(i,0,k)\,[S_1^i z_3^{-k}]x \\
&+\textstyle\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{k=-K}^{K} h(i,j,k)\,[S_1^i(z_2^{j}+z_2^{-j})z_3^{-k}]x
\end{aligned}
\tag{12}
$$

Application of the Central Sum Operators of Eqn (8) reduce this equation to

$$
\begin{aligned}
y = &\textstyle\sum_{k=-K}^{K} h(0,0,k)\,[z_3^{-k}]x \\
&+\textstyle\sum_{j=1}^{K}\sum_{k=-K}^{K} h(0,j,k)\,[S_2^j z_3^{-k}]x \\
&+\textstyle\sum_{i=1}^{K}\sum_{k=-K}^{K} h(i,0,k)\,[S_1^i z_3^{-k}]x \\
&+\textstyle\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{k=-K}^{K} h(i,j,k)\,[S_1^i S_2^j z_3^{-k}]x
\end{aligned}
\tag{13}
$$

Examination of the limits of the above equation shows that the number of multiplications has been reduced to $(2K+1)(K+1)^2$ allowing for symmetry about two axial planes.

We now consider the remaining axial plane k=0 for which even symmetry implies:

$$h(i,j,k) = h(i,j,-k) \tag{14}$$

Separating into positive, zero and negative components in 'k' as before and proceeding similarly ultimately yields:

$$
\begin{aligned}
y = \; & h(0,0,0)x \\
& + \sum_{i=1}^{K} h(i,0,0)\,[S_1^i]x \\
& + \sum_{j=1}^{K} h(0,j,0)\,[S_2^j]x \\
& + \sum_{k=1}^{K} h(0,0,k)\,[S_3^k]x \\
& + \sum_{j=1}^{K}\sum_{k=1}^{K} h(0,j,k)\,[S_2^j S_3^k]x \\
& + \sum_{i=1}^{K}\sum_{k=1}^{K} h(i,0,k)\,[S_1^i S_3^k]x \\
& + \sum_{i=1}^{K}\sum_{j=1}^{K} h(i,j,0)\,[S_1^i S_2^j]x \\
& + \sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{k=1}^{K} h(i,j,k)\,[S_1^i S_2^j S_3^k]x
\end{aligned}
\tag{15}
$$

Eq(15) is the reduced form of the convolution for complete axial symmetry in 3D. The number of multiplications required to compute the convolution in this reduced form is just $(1+K)^3$ which for higher order K is an eight-fold speed up over the general expression of Eq(1), namely $(2K+1)^3$.

3 Diagonal Symmetry

By diagonal symmetry symmetry we mean (in 3D) symmetry about the six diagonal planes $i=j$, $i=-j$, $i=k$, $i=-k$, $j=k$ and $j=-k$. We restrict consideration to filters which possess even axial symmetry, so that diagonal symmetry over the three 'positive' diagonal planes, $i=j$, $i=k$, and $j=k$ only needs to be considered.

To determine the form for full axial symmetry and symmetry about the single diagonal plane i=j, we separate Eq(15) into its axial, diagonal and off-axis/ off-diagonal components. Exchanging 'i' and 'j' in the terms affected by the reflection gives.

$$
\begin{aligned}
y = \; & h(0,0,0)x \\
& + \sum_{i=1}^{K} h(i,0,0)\,[S_1^i]x \\
& + \sum_{i=1}^{K} h(0,i,0)\,[S_2^i]x \\
& + \sum_{k=1}^{K} h(0,0,k)\,[S_3^k]x \\
& + \sum_{i=1}^{K}\sum_{k=1}^{K} h(0,i,k)\,[S_2^i S_3^k]x \\
& + \sum_{i=1}^{K}\sum_{k=1}^{K} h(i,0,k)\,[S_1^i S_3^k]x \\
& + \sum_{i=2}^{K}\sum_{j=1}^{i-1} h(i,j,0)\,[S_1^i S_2^j]x \\
& + \sum_{i=1}^{K} h(i,i,0)\,[S_1^i S_2^i]x \\
& + \sum_{j=1}^{K}\sum_{i=j+1}^{K} h(j,i,0)\,[S_1^j S_2^i]x \\
& + \sum_{i=1}^{K}\sum_{k=1}^{K} h(i,i,k)\,[S_1^i S_2^i S_3^k]x \\
& + \sum_{i=2}^{K}\sum_{j=1}^{i-1}\sum_{k=1}^{K} h(i,j,k)\,[S_1^i S_2^j S_3^k]x \\
& + \sum_{j=1}^{K}\sum_{i=j+1}^{K}\sum_{k=1}^{K} h(i,j,k)\,[S_1^j S_2^i S_3^k]x
\end{aligned}
\tag{16}
$$

However, diagonal symmetry about i=j implies that

$$h(i,j,k) = h(j,i,k) \tag{17}$$

Applying this relation to the reflected terms of the above equation and changing the order of summation of these terms gives.

$$
\begin{aligned}
y = \; & h(0,0,0)x \\
& + \textstyle\sum_{i=1}^{K} h(i,0,0)\,[S_1^i + S_2^i]x \\
& + \textstyle\sum_{k=1}^{K} h(0,0,k)\,[S_3^k]x \\
& + \textstyle\sum_{i=1}^{K}\sum_{k=1}^{K} h(i,0,k)\,[S_1^i S_3^k + S_2^i S_3^k]x \\
& + \textstyle\sum_{i=2}^{K}\sum_{j=1}^{i-1} h(i,j,0)\,[S_1^i S_2^j + S_1^j S_2^i]x \\
& + \textstyle\sum_{i=1}^{K} h(i,i,0)\,[S_1^i S_2^i]x \\
& + \textstyle\sum_{i=1}^{K}\sum_{k=1}^{K} h(i,i,k)\,[S_1^i S_2^i S_3^k]x \\
& + \textstyle\sum_{i=2}^{K}\sum_{j=1}^{i-1}\sum_{k=1}^{K} h(i,j,k)\,[(S_1^i S_2^j + S_1^j S_2^i)S_3^k]x
\end{aligned}
\tag{18}
$$

The number of multiplications involved in the evaluation of this symmetry reduced convolution is $\frac{1}{2}(K^3 + 4K^2 + 5K + 2)$. That is, about a 50% reduction. We have shown how, starting from the reduced form of the convolution with full axial symmetry in Eq(15), how to take into account a further symmetry, namely diagonal symmetry in the $k = 0$ plane. Proceeding in the same manner, to incorporate the effect of diagonal symmetry in the $i = 0$ and $j = 0$ planes, leads to the following expression for the symmetry reduced convolution:

$$
\begin{aligned}
y = h(0,0,0)x & + \sum_{i=1}^{K} h(i,0,0)\,[S_1^i + S_2^i + S_3^i]x + \sum_{i=1}^{K} h(i,i,0)\,[S_1^i S_2^i + S_1^i S_3^i + S_2^i S_3^i]x \\
& + \sum_{i=2}^{K}\sum_{j=1}^{i-1} h(i,j,0)\,[S_1^i S_2^j + S_1^j S_2^i + (S_1^i + S_2^i)S_3^j + (S_1^j + S_2^j)S_3^i]x \\
& + \sum_{i=2}^{K}\sum_{k=1}^{i-1} h(i,i,k)\,[S_1^i S_2^i S_3^k + S_1^k S_2^k S_3^i + (S_1^i S_2^k + S_1^k S_2^i)S_3^i]x \\
& + \sum_{i=1}^{K} h(i,i,i)\,[S_1^i S_2^i S_3^i]x + \sum_{i=2}^{K}\sum_{j=1}^{i-1} h(i,j,j)\,[(S_1^i S_2^j + S_1^j S_2^i)S_3^j]x \\
+ \sum_{i=3}^{K}\sum_{j=1}^{i-1}\sum_{k=1}^{j-1} & h(i,j,k)\,[(S_1^k S_2^j + S_1^j S_2^k)S_3^i + (S_1^i S_2^k + S_1^k S_2^i)S_3^j + (S_1^i S_2^j + S_1^j S_2^i)S_3^k]x
\end{aligned}
\tag{19}
$$

The number of multiplications involved in the evaluation of the last expression is $\frac{1}{6}k^3 + k^2 + \frac{11}{6}k + 1$ This represents, for large filter sizes, a six-fold improvement over the result obtained for axial symmetry (only).

Conclusions

We have analysed the implications of both axial and diagonal symmetry for a general three dimensional convolution. Our calculation has been expressed in terms of shift operators, z_1,z_2,z_3 which generate translations along coordinate axes. The final forms obtained, which might be termed symmetry reduced convolution formulas, have been further simplified by the use of the central sum operators, S_1^a etc as per Eq(8) . With increasing filter order, the full utilization of axial symmetry leads to an eight-fold reduction of the number of multiplications needed to perform a convolution. Diagonal symmetry, by itself, was not calculated. We have shown that once axial symmetry has been utilised, the further reduction in the number of muliplications due to diagonal symmetry is by a factor that tends to six as the filter order increases. Overall the combined effect of axial and diagonal symmetry is to reduce the number of multiplications by a factor that tends to 48 for high filter orders.

For comparision, in 2-D, for which implications of symmetry have been mentioned by Mersereau [6], the effect of axial symmetry is to effect a four-fold reduction, while axial plus diagonal symmetry together effect an eight-fold reduction. It is clear that with increasing filter dimensions the computational savings will be even more significant.

It must be mentioned that if a filter is factorizable, there is significant computational advantage in replacing the higher order filter by a product of lower order filters. The prime example of a factorizable filter is the gaussian. However many important filters possess a great deal of symmetry, but are not decomposible; for instance the rho filter of tomography, [5] [4] spherical equalizers of odd-powers of rho, band pass filters [1].

Due to the computational expense of evaluating convolutions for large filter orders, these tend to be replaced by fourier domain processing utilizing the FFT algorithm. Computational savings due to redundancy in the filter structure shift the crossover point from convolutions to fourier processing to higher filter orders.

The symmetry reduced expressions that we have derived are in a suitable form for direct programming. However these expressions are also especially convenient in the design of a systolic array [7] or a pipe-line processor. A further area of application is in the design of active sensors [8] such as that of Nishizawa et al [9] where the initial sensing is combined with convolution operations in a single chip.

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