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Texture Energy Tensor for Texture Discrimination

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A new, computationally inexpensive, approach to texture recognition and segmentation is proposed based on a discriminating quantity called the <u>texture energy tensor</u> (TET). The TET is defined as the sum over a window of the products of pixel pairs at fixed vector displacement. For mathematical textures the TET gives easy to interpret results. Texture classification and segmentation using Laws masks or "tuned masks" and "texture energy" is shown to be a special case of a TET approach.

Introduction

The segmentation of natural scenes, of biological images, of manufacturing scenes involving agricultural produce require the further development of techniques for texture analysis. The simplest approach, determining gray scale averages over small windows, has limited applicability. Early approaches using fourier analysis were not markedly successful. Later more sophisticated methods were developed such as the use of co-occurrence matrices [1] which while serviceable have excessive computational cost. About 1980 Laws [4], see also [5], developed a computationally modest means for the discrimination, recognition and segmentation of textures. In Laws' approach, the texture is first convolved with one of several specific masks, then what Laws termed the *texture energy* is calculated as the average square of the (resultant) pixel (for a suitably sized window). The method of Laws is analysed, and it is shown that the various *texture energies* associated with the different masks may be well replaced by the calculation of the components of the <u>texture energy tensor.</u> The particular advantage of using this new quantity is a significant savings in computation compared to texture discrimination using multiple masks.

Recently Skinner and Benke [6] and Cohen and You [7] suggested the use of ***texture tuned*** *masks* masks similar to the Laws masks, to be used in texture energy calculations, but with mask elements which maximise the *texture energy* of a specific texture. Thus whereas particular Laws masks are more spot sensitive or more responsive to edges or lines, such a tuned mask serves to highlight a specific texture. In the approaches of Benke, and in that of Cohen and You, tuned masks were determined by a computationally expensive optimisation procedure. Here it is shown that *texture tuned masks* are directly related to the <u>texure energy tensor,</u> which is far more convenient to compute.

The texture energy tensor

The <u>texure energy tensor</u> (TET) is defined for an M*N window over a digitised image I by the formula:

$$T_{r,s}(u,v)\ \ \frac{1}{MN}\sum_{i,j} I_{i,j}\, I_{i+r,j+s}$$

Where the sum is over all (i, j) pixels in the window centred on the pixel (u,v). More precisely, if M = 2m+1, and N = 2n +1, one has:

$$T_{r,s}(u,v) = \sum_{1=-n}^{1=n}\ \sum_{i=-m}^{1=m} I_{u+i,v+j}\, I_{u+i+r,v+j+s}$$

Note that the TET components can be defined at any pixel location (1,m) sufficiently) far from the image boundary.

When evaluated for a window contained within *a* larger region of uniform texture $T_{r,,s} = T_{-r,-s}$

Note that T_{00} is simply the autocorrellation within the window.

In a purely man-made texture that is strict1y periodic with every (r, s) then

$$T_{T,S} = T_{00}$$

and is a maxima compared to other components of the TET.

Examples using Ideal Textures

It is interesting to calculate the TET for mathematical textures. For example, consider a texture comprised of vertical stripes of period 5, with two black stripes, of gray scale 0, then 3 white stripes of gray scale 1.

1 1 1 0 0 1 1 1 0 0 1 1 1 0 0

1 1 1 0 0 1 1 1 0 0 1 1 1 0 0
1 1 1 0 0 1 1 1 0 0 1 1 1 0 0
1 1 1 0 0 1 1 1 0 0 1 1 1 0 0

Over a window size MN with M a multiple of five, one has exactly for this "texture":

$T_{0j} = 3/5$ **for all j**

$T_{1j} = 2/5$ **for all j**

$T_{2j} = 1/5$ **for all j**

$T_{3j} = 1/5$ **for all j**

$T_{4j} = 2/5$ **for all j**

$T_{5j} = 3/5$ **for all j**

This artificial texture is "uniform" in the sense that the TET component values are independent of location. In contrast, for the similar "uniform" texture with a slant:

1 1 1 0 0 1 1 1 0 0 1 1 1 0 0

1 1 0 0 1 1 1 0 0 1 1 1 0 0 1
1 0 0 1 1 1 0 0 1 1 1 0 0 1 1
0 0 1 1 1 0 0 1 1 1 0 0 1 1 1

the texture energy tensor has components, for evaluation over MN window with M a multiple of five:

$T_{00} = 3/5$

$T_{10} = 2/5$

$T_{11} = 3/5$

$T_{1,-1} = 1/5$

For the third variant:

1 1 1 0 0 1 1 1 0 0 1 1 1 0 0

1 0 0 1 1 1 0 0 1 1 1 0 0 1 1
0 1 1 1 0 0 1 1 1 0 0 1 1 1 0
1 1 0 0 1 1 1 0 0 1 1 1 0 0 1

$T_{00} = 3/5$

$T_{10} = 2/5$

$T_{11} = 1/5$

$T_{1,-1} = 1/5$

Thus $T_{1,-1}$ is a simple, and natural, discriminant of these three textures.

Relationship to Laws Method

A basic step in Laws procedure is the replacement of an image appearing in an N*M window by the result of convolving with one of certain masks (3*3 or 5*5): **I** ----->J= **A * I**

where in the 5*5 case

$$A = \begin{pmatrix} a_{-2,-2} & a_{-2,-1} & a_{-2,0} & a_{-2,1} & a_{-2,2} \\ a_{-1,-2} & a_{-1,-1} & a_{-1,0} & a_{-1,1} & a_{-1,2} \\ a_{0,-2} & a_{0,-1} & a_{0,0} & a_{0,1} & a_{0,2} \\ a_{1,-2} & a_{1,-1} & a_{1,0} & a_{1,1} & a_{1,2} \\ a_{2,-2} & a_{2,-1} & a_{2,0} & a_{2,1} & a_{2,2} \end{pmatrix}$$

(The novel index labelling of convolution masks utilised in this paper was adopted to simplify various expressions.)
In terms of components, one has

$$J_{rs} = \sum_i{}' \sum_j{}' a_{i,j} I_{r+i,s+j}$$

Where the Σ' denotes a sum over convolution mask index range, whereas the unprimed Σ is used here strictly to denote a window range sum.

What Laws terms the *texture energy* is calculated over an M*N window as

$$\frac{1}{MN}\sum_r \sum_s J_{r,s} J_{r,s}$$

Note that the window sum is over the elements of the convolved image. The *texture energy, is* a quadratic expression in the mask elements. In this quadratic sum the coefficient of $(a_{0,0})^2$ is simply the window sum

$$T_{0,0} = \frac{1}{MN}\sum_{i,j} I_{ij} I_{ij}$$

and is also equal to the texture energy of image before masking. If the window is located in a region of uniform texture, and is large enough, so that sliding the window by a few pixels will not alter the sums, it is apparent that the coefficient of any term $(a_{i,j})^2$ has the same value $T_{0,0}$.

More generally, for a sufficiently large window located in a larger region of uniform texture, the coefficient of $a_{i,j}\, a_{i+r},$ is given by

$$T_{r,s} = \frac{1}{MN}\sum_{i,j} I_{i,j} I_{i+r,j+s}$$

that is, by an element of the **texure energy tensor**

Detailed Analysis of Laws

In this section a detailed derivation is presented of the result presented in the previous section.

In full, the Laws texture energy over M*N window is

$$\frac{1}{MN}\sum_r \sum_s J_{rs} J_{rs}$$

where the J_{rs} is the convolved image,

$$J_{rs} = \sum{}' \sum{}' a_{i,j} I_{r+i,s+j}$$

Where the Σ' denotes a sum over convolution mask index range, whereas the unprimed Σ is used here strictly to denote a window range sum. Expanding the TET, one has, apart from the scale factor of (1/MN):

$$\sum_r \sum_s \sum_i{}' \sum_j{}' a_{i,j} I_{r+i,s+j} \sum_k{}' \sum_l{}' a_{k,l} I_{r+k,s+l}$$

Interchanging the order of summation one has:

$$\sum_i{}' \sum_j{}' \sum_k{}' \sum_l{}' a_{i,j}\, a_{k,l} \sum_r \sum_s I_{r+i,s+j}\, I_{r+k,s+l}$$

Ir+k

which is (replacing the scale factor)

$$\frac{1}{MN}\sum_i{}' \sum_j{}' \sum_k{}' \sum_l{}' a_{i,j}\, a_{k,l}\, T_{k-i,l-j}\, (i,j)$$

This final result is the expression of Law's texture energy , for a texture convolved with mask A, in terms of the texture energy tensor introduced in this paper. Note shift of location of the window utilised by the TET, which is not consequential during training stages, but does bear on edge location precision.

Connection with Tuned Discrimination Masks

Both Skinner and Benke [6] and Cohen and You [7] used stochastic methods to determine masks that maximised Law's texture energy. However, using the last equation, it is clear that a much simpler computational method is available. The key idea is that the (Law's) texture energy is simply a quadratic form on the texture energy tensor.

To compute a M*M tuned convolution mask which after application to a given texture, gives maximum (Laws) texture energy, the following algorithm suffices: Determine the eigenvectors of the corresponding M*M components of $T_{r,s}$. The eigenvector corresponding to the minimum eigenvector consists of the components of the 'tuned' mask. This eigenvector is determined to a scale factor by standard methods.

Discussion

The texture energy tensor (TET), unlike other tools for texture analysis, is especially attuned to local repetition of gray scale features - but only at the same scale.

The analysis scheme presented here involves the determination of standard texture energy tensor by calculation over a large window of uniform texture. Discrimination and segmentation involve the utilisation of smaller windows, 15*15 to 17*17. Where a number of textures are involved, statistical clustering methods similar to those utilised by Laws [4] will be required to determine linear combination of tensor elements most appropriate for discrimination between specified textures.

The traditional categorization in texture discrimination is between structural and statistical approaches. The texture energy approach introduced here falls astride this dichotomy, having both simple structural aspects, as especially shown with idealised textures, as well as being inherently statistical, involving as it does what is really an autocorrellation function defined on a window.

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