Decoding of Block-Oriented Fractal Image Coding: Design Issues

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ABSTRACT: *This paper is concerned with an improved algorithm called the scan algorithm for the decoding of block-oriented fractal encoded images. The decode algorithm as used by Jacquin and also by Fisher et al is here called the "2-generation" algorithm. Examples of the decoding of realistic gray-scale images are given that indicate that the 'scanning .algorithm", is ,found to be about 40% faster than the 2-generation algorithm.*

1 INTRODUCTION

Following the pioneering work of Williams [1] and Hutchinson [2] on the synthesis of deterministic fractals, Barnsley and co-workers [3][4][5] further proposed that fractal encoding could be applicable, and effective for the compressive encoding of gray-scale and high quality colour images. Despite the demonstration by Barnsley of some extraordinary high image compressions using fractal. compression, using 'hand-encoding' [5], it was for some years unclear how an automatic fractal encoding system would function, However, following the patent [8] granted to Michael Barnsley for the fractal encoding of images, a practical scheme for the block-oriented encoding of an arbitrary gray-scale image has been given by Jacquin [6][7]. Following Jacquin, other. workers, notably Fisher [8][9] have implemented functional schemes for block-oriented fractal encoding. Although the actual compressions reported for individual (still) images using fractal coding do not at this date match those of DCT-based algorithms, there is arguably superior subjective image quality for the fractal-based images, and significant commercial application has already taken place, notably with the images of the Microsoft "Encata" CD Rom Encyclopaedia. There is considerable scope for incorporating fractal coding in the emerging low-hit rate video codecs.

Jacquin [6] [7] and Fisher et al [8][9] have shown how 2-level and quad-tree coding can improve the compression efficiency. Monro et al [12] have shown that a simplified transform called the Bath Fractal Transform offers significant encoding and compression advantages. The improved decoding algorithm, the scanning algorithm described here is a generalisation of the algorithm of Cohen [10] for synthesising IFS fractals.

The plan of the paper is to first describe and compare the 2-generation and the scanning decoding algorithms, which are analogous to Jacobi and Gauss-Seidel iterative schemes (respectively). Then we 'present experimental results comparing the two decode algorithms as applied to images that have been coded using range blocks of fixed size 4x4.

2. BLOCK ORIENTED FRACTAL CODE

2.1 Theoretical Basis for Binary Images

The basis for fractal coding is that a digitised image . is approximated by a fractal which is the attractor of a set of contractive mappings of the plane. For the case of binary images the simplest such fractal coding is of the form termed by Barnsley [3] IFS (Iterated Function Systems) and involves a set S of N maps W;

$$S = \{ W_1, W_2, W_3, \ . \ . \ . \quad , W_N \}$$

where each map has a contractivity less than 1. The attractor A, which in this context is the fractal that is the digital approximation to the binary set, is the union of copies of itself:

$$A = W_1(A) \cup W_2(A) \cup \ .. \ . \cup W_N(A)$$

For a binary image, coding involves finding the set of N maps such that the difference d between the image and the union of transformed copies of itself is minimised:

$$d = \ \|I - \mathrm{W1}(\mathrm{A}) \cup \ldots \|$$

The set to set distance used in theoretical discussions is the Hausdorff metric [16] . Barnsley et al [4] showed that the error in the attractor is then $\|I - A\| < d/(1-s)$ where s is the maximum contractivity of the N maps. This basic result is known as the Collage Theorem.[16]

2.2 Block-Oriental Fractal Decoding

For IFS encoded_ (binary) image sets the code is the set of mappings in the IFS set. For comparison purposes we note that in IFS coding the mappings have as their domain a region larger than the marked pixels of the image set (attractor), while the attractor, or more precisely each map W_r of the IFS set S has the range set W_r (I). In block-oriented fractal coding, as introduced by Jacquin [6], a narrower conception of similarity is used, which involves the similarity of one small portion of the image with a larger portion. To code an image, involves segmenting an image into disjoint blocks, called range blocks, and ascribing to each block a transformation that relates the pixel gray-scales in that block to the pixels in larger block "domain" block which is taken to be four times the size of the "range" block. That each transform has a given domain and smaller range that are both sub-sets of the image. The set of transformations for all the domain blocks of the image constitute the fractal code for the image. Fig (I), although directed at decoding, also shows this scheme for a fixed partitioning of an image into 4x4 range blocks, with 8x8 domain blocks located anywhere in the image. In this paper we consider for each block the set of 8 possible combinations of a 90 degree rotation with an optional reflection.

Encoding involves locating in the image the domain block that best matches each range block. As for each of the 64x64 = 4096 disjoint 4x4 range block in a 256x256 image there are 8 rotated/reflected forms of each of the 249x249 different possible domain blocks this is a computationally expensive process. We follow Fisher et al, and associate with each range block a linear transform, with contrast s_i and luminance so, and use the rims error of each range block as the error condition. That, we seek to minimise the sum

$$\Sigma (p, - s_i \ * q_i \ + \ so)^2$$

where the sum is over all the pixels of gray-scale p in the range-block, the gray-scale in the compressed and transformed domain block being q.

2.3 Decoding Algorithms

In Jacquin's papers [6] [7] , and in the work of Fisher et al [8][9], the decoding of block-oriented fractal coded images has been implemented as follows:

Take an arbitrary starting image as iteration 0., Compute each range block of iteration i (>=1) using the corresponding transformation. applied to a domain block located in the image of iteration i -1.

(Jacquin also indicates use of a code-book, which does not affect this discussion.) The order in which the range blocks are computed is of no consequence. This algorithm is here termed the '2-generation' algorithm.

In this paper, a new decode algorithm called the 'scanning algorithm' is introduced. The algorithm is as follows:

Take an arbitrary starting image as scan 0. Compute each range block of image scan number i (>=1) using the corresponding transformation. applied to a domain block located in the very same image that is being upgraded (range) block by block during the scan.

The two scanning algorithms are further explained in Fig (i) and Fig (ii).

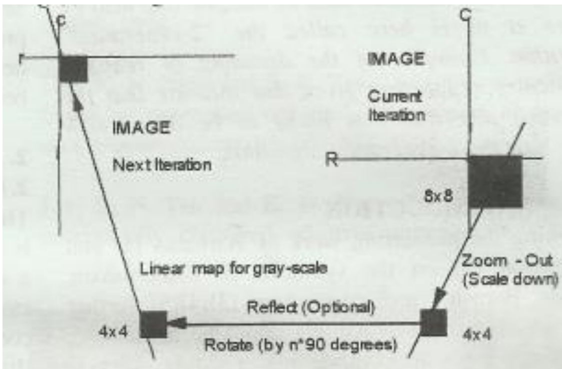
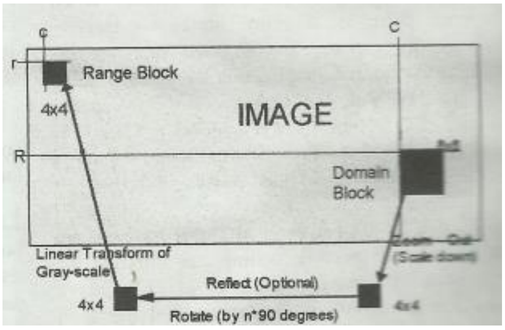


Fig (i) The 2- Generation Decoding Algorithm. (For the case of the segmentation of an image into 4x4 disjoint range blocks; no code-book; linear gray-scale mapping.) In the next generation image. each (new) range block is derived from the specified 8x8 domain block by compressing to 4x4 size, minting_ through a multiple of 90 degress, and **optionally reflecting** about a centre-line; the actual **gray-scale of eat** pixel in the range block being then derived **by a** linear transform, the same contrast and **luminance** being applicable for each pixel in the range block_ The 2-generation algorithm involves repetitively applying the transformations of the fractal code until convergence



Fig(ii) The Scanning Decoding Algorithm. (For the case of the segmentation of an image into 4X4 disjoint range blocks: linear gray-scale mapping.) In each image scan the image is scanned, in this study in a TV raster manner, and each range block in the image is replaced - ie updated using the transform code (mapping) for that block, applied to the specified domain block that is within the very same image. The image scans can be repeated until convergence

The formal proof of convergence of fractal decode algorithms remains an area of research [15]. The more significant question as to the rate of convergence of the decode algorithm has not been the subject of any theoretical analysis. In this paper the effectiveness of two competing decode algorithms is determined experimentally.

3 EXPERIMENTAL STUDY

In this paper we compare the 2-generation decode algorithm with the scanning decode algebra with respect to the decoding of a gray-scale Lena image. In this case the fractal code was for 4x4 range blocks, with 8x8 domain blocks situated anywhere in the image, what Fisher [9] terms 1 pixel inter-block with respect to the family of candidate blocks to be matched.

For the decode of Lena (256x256) with starting point gray-scale Mandrill, a side-by-side comparison of the 2-generation decode algorithm versus the new scanning algorithm is tabled below. Some images indicating the progress of decoding according to the new algorithm are presented in Fig (3) below.

To trace the convergence of the decode iteration we have for each iteration computed the Signal to Noise ration SNR of the image with respect to the original image (before encoding).

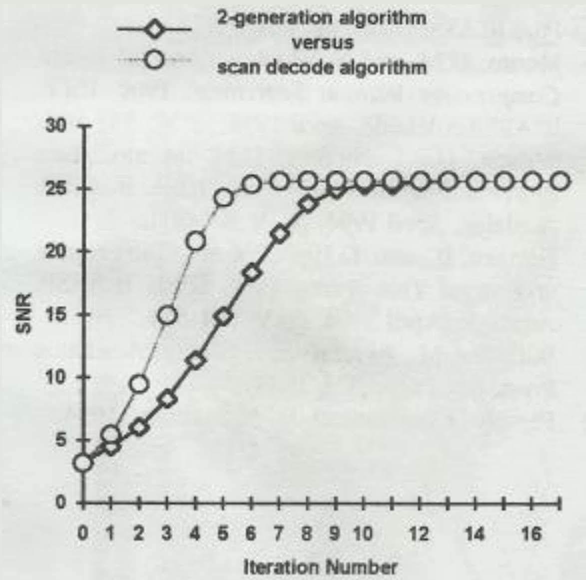


Fig (iii) Plot of SNR/Iteration for conventional 2-generation generation algorithm compared to the scanning algorithm. SNR is on rms error basis with respect to the lena image before coding. In both cases the initial image was a monochrome mandrill image**.** Note that for the scanning image a single iteration is taken to be a total scan of the image, updating en route. The same table is presented in the table below.

TABLE: DECODE DATA

Iteration No Scan No Initial Image mandrill	SNR at end of scan distinct 2- generation Algorithm dB	SNR at end of scanning algorithm dB
0	3.1694	3.1694
1	4.4741	5.4312
2	6.0470	9.4863
3	8.3506	14.9607
4	11.4008	20.8297
5	14.9027	24.2900
6	18.3981	25.4037
7	21.4589	25.6167
8	23.8152	25.6670
9	24.9549	25.6542
10	25.3545	25.6542
11	25.3545	25.6603
12	25.6193	25.6504
13	25.6284	25.6501
14	25.6470	25.6461
15	25.6477	25.6556
16	25.6614	25.6522
17	25.6555	25.6573

The tabled results show that an SNR of better than 20 dB is reached after only 4 iterations of the scanning algorithm, whereas the 2-generation algorithm requires 7 iterations to reach that image fidelity. The fidelity level of 25 dB is reached after 6 scans for the scanning algorithm, versus 10 iterations for the 2-generation. We note that one scan of the scanning algorithm is marginally faster than 1 iteration of the 2-generation algorithm. Hence, from this example, and others, decode speed-up of the order of 40% is established. This conclusion is graphically demonstrated by tracing through the convergence of the two decode algorithms, as given in Fig (iv).

4. CONCLUSIONS

Fractal coded for images is already a commercial reality: the extent to which fractal coding can supplant or complement DCT-based coding will only become fully apparent after a period of development. In this paper, a very basic feature of fractal encoding, the iterative algorithm used for decoding has been examined.

The algorithm used in the first paper on block-oriented fractal encoding, that of Jacquin, has been here termed the 2-generation decode algorithm. A new algorithm, called the scan decode algorithm for block-oriented fractal coding has been presented. Data has been given for a typical image, that of Lena, comparing the speed of the decode process using the 2-generation algorithm, compared **to** the new algorithm. The new algorithm, that is, the scanning decode algorithm, has been shown to be of the order of 40% faster than the 2-generation algorithm. Although only the case of fixed partitioning has been investigated, it is suggestive that speed-ups of similar order will occur for quad-tree and other adaptive coding. This result is of significance both for the fractal decoding of still images, as well as for the emerging variety of fractal coded video, such as that developed by Monro et al at Bath, [141, and that just recently developed by Iterated Systems [17].

An appreciation of the new algorithm can be obtained from the recognition that the fractal coding of gray-scale images involves a generalisation of the schemes used for the fractal coding of binary images, notably the Iterated Function System.(IFS). This writer has previously shown that scan-type algorithms significantly speeded up the decode process (image synthesis) for IFS. [1011111.

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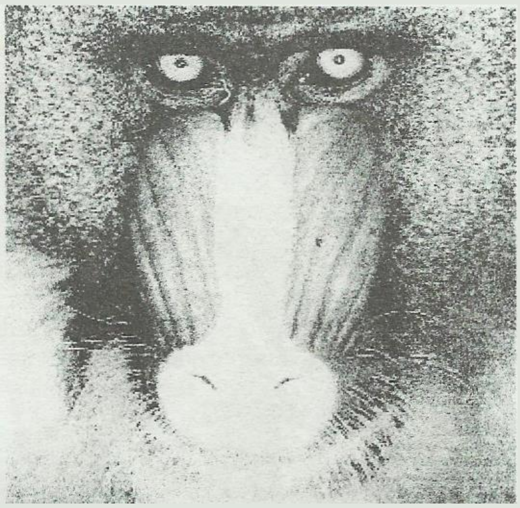
Fig (iv) Comparison of the previously applied algorithm, the 2-generation decode algorithm, with the scanning decode algorithm.

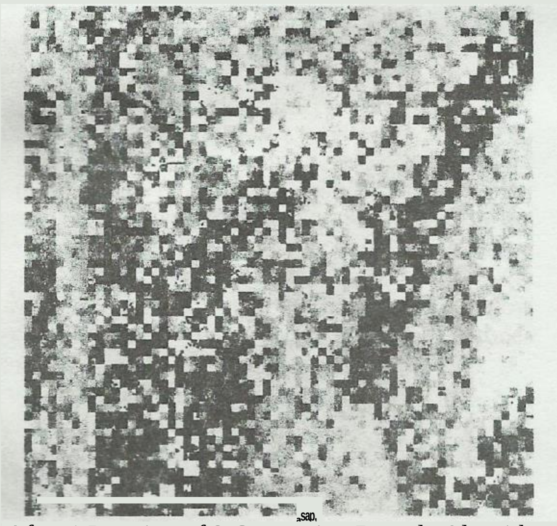
For this comparison, the initial image in both cases is an image of Mandrill.

The scanning decode algorithm was applied in raster scan of the range blocks. starting at the top left corner, and using the (partly) updated image as a source of range blocks en route.

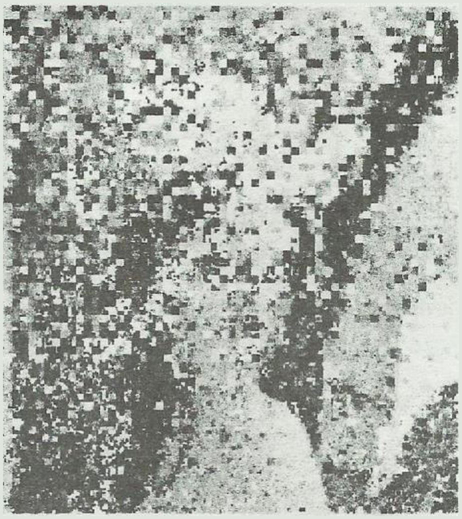
The signal to noise ratio SNR is calculated on the basis of the ratio of rms signal difference to nns of lena image used for encoding. The encoding was performed using 4x4 range blocks. with 8x8 domain blocks located any-where in the image.



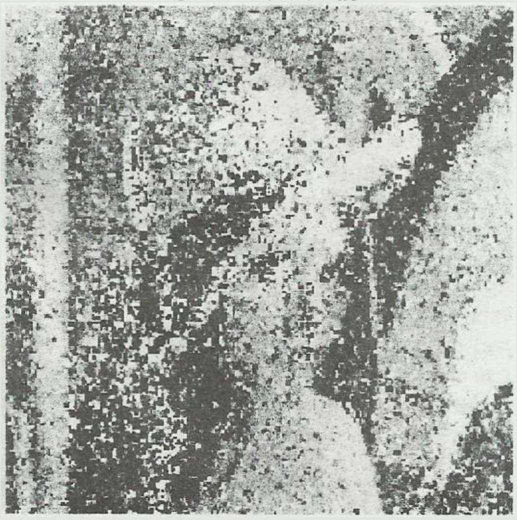
Initial Image (Mandrill)
SNR = 3.1694



After 1 Iteration of 2 Generation Decode Algorithm
SNR = 4.4741 db



After 1 Iteration of Scanning Decode Algorithm
SNR = 5.4312



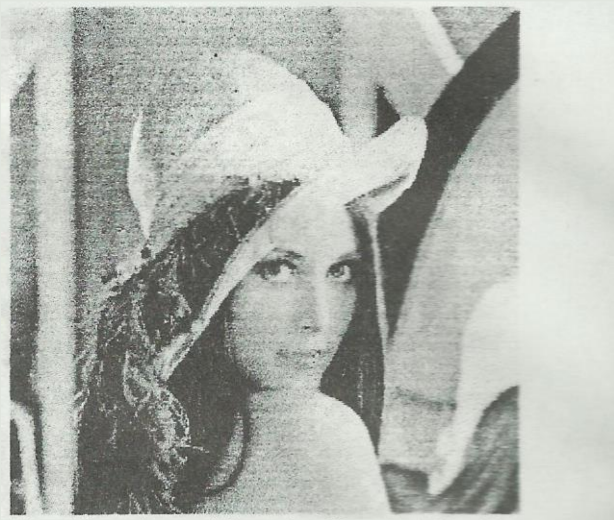
After 2 Iterations of 2 Generation Decode Algorithm
SNR = 6.0470

After 2 Iterations of Scanning Decode Algorithm
SNR = 9.4863

- After 3 Iterations of 2 Generation Decode Algorithm SNR = 8.3431

After 3 Iterations of Scanning Decode Algorithm SNR = 15.0028

After 4 Iterations of 2 Generation Decode Algorithm SNR = 11.4012



After 4 Iterations of Scanning Decode Algorithm SNR = 20.9989

After 5 Iterations of 2 Generation Decode Algorithm SNR = 14.9377

After 5 Iterations of Scanning Decode SNR = 24.7520